THE PUPILS' ARITHMETIC

BOOK THREE

BYRNES-RICHMAN-ROBERTS

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BY

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PREFACE

PART THREE of *The Pupils' Arithmetic* contains a review of the fundamental rules, a full treatment of common fractions and decimal fractions, a brief treatment of the reduction of denominate numbers and the mensuration of rectilinear figures, and an introduction to percentage. Its scope, therefore, is sufficiently broad to cover the work in arithmetic of the fifth school year everywhere in the United States, and to embrace part of the work of the sixth year in those school systems which prescribe a less extensive range of topics for the fifth year.

Part One and Part Two of the series were so well received that the authors have been emboldened to extend and to emphasize in Part Three those features which gave distinction to the earlier volumes. Not at the first glance, it is true, are these features always appreciated. But with each day's use, the advantages of a text designed to meet every phase of the lesson become apparent; and the book becomes what it was intended to be — a text for reading and discussion, an exposition of principles, an exhibit of model forms, and a

storehouse of drills, sight exercises, oral exercises, manual exercises, and problems ready for immediate class use.

The book embodies and illustrates methods of teaching which have stood the tests of classroom practice and of theoretical criticism from both mathematical and pedagogical standpoints.

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BOOK THREE

PUPILS' ARITHMETIC

BOOK III

REVIEW OF FUNDAMENTAL OPERATIONS

ADDITION

Types of Oral Drills for Frequent Practice

1. 1. Add by 7's to the number nearest 100. What is the number?

2. Add by 8's from 3 to the first total beyond 100. What is that total?

3. Beginning with 5, add 9 at each tap of the pencil.

4. Find the sum of: 21, 8, 3, 6, 10, 5, 8, 11.

5. What is the amount of: 25 ct., 16 ct., 10 ct., 27 ct., 5 ct. ?

6. Add 12 to each of the following: 18, 21, 35, 27, 39, 43, 56.

SIGHT EXERCISES

For Oral Drill From Text, Blackboard, or Chart, or for Seat Work in Limited Time

2. Add 38 and 26.

Hint. Add thus, 38, 58, 64.

Give sums quickly:

1.
16	14	17	18	19	11	13	18
13	12	14	15	17	17	19	19

2.
23	27	25	28	23	29	21	26
19	15	17	20	25	24	27	26

3.
35	38	36	33	39	31	34	32
10	26	36	28	35	48	37	49

4.
46	49	51	57	53	29	54	48
23	28	19	26	36	57	39	35

5.
67	74	53	87	93	63	76	89	72
41	38	29	27	14	48	47	29	86

Read each example and give the answer quickly:

6. $35 + 27 = ?$ $68 + 37 = ?$ $49 + 73 = ?$ $28 + 43 = ?$

7. $18 + 58 = ?$ $34 + 57 = ?$ $29 + 92 = ?$ $53 + 49 = ?$

Add:

8.
$ 50	67 ct.	54 lb.	27 ft.	58 min.	16 qt.
25	32 ct.	28 lb.	38 ft.	70 min.	49 qt.
10	9 ct.	12 lb.	5 ft.	25 min.	33 qt.

Add rapidly, both columns at once:

9.	8	10.	7	11.	4	12.	9	13.	8
	3		9		10		4		10
	11		5		8		10		3
	7		10		9		11		3
	10		8		27		6		14
	14		26		2		51		16

14.	13	15.	12	16.	18	17.	25	18.	41
	7		5		12		34		34
	20		13		24		41		25
	16		16		23		19		19
	4		14		13		28		63
	18		8		15		42		27

Prove Examples 9, 11, 15, 18.

WRITTEN EXERCISES

3. Add, looking for combinations of 10.

Verify each answer by adding in reverse order.

1.	29	2.	645	3.	5208	4.	$ 5.50	5.	$21.46
	31		328		1974		17.25		3.94
	46		165		677		.68		49.27
	33		256		5831		43.51		28.31
	24		184		6024		95.12		47.56
	18		207		7935		13.69		29.39
	42		902		2896		24.78		106.74

Verify each answer of the following examples by adding the columns separately (without carrying) and then adding the sums, as in example 6:

	6.	7.	8.	9.
	9604	25476	47798	$ 176.48
	3210	16934	62243	297.33
	745	28109	49396	1564.90
	651	74261	82921	8.75
	2292	3905	50007	190.83
	4378	2666	29315	3152.71
	20	70050	46127	74.95
	26	753	69389	695.14
	26	14287	16728	4927.36
	18	27644	43125	8191.82
	20880			

10.	11.	12.	13.
454692	1250643	2987600	3008316
98847	3974792	5246325	597289
746631	750475	6060209	7136184
47770	1293628	4004050	2613585
633495	2471803	569293	3749126
387695	3008129	8427358	6095317

14.	15.	16.	17.
57195	28416	194371	214726
83942	37628	209315	531274
21746	19208	642537	618598
92813	50725	483161	293639
53129	86427	920740	515426
69318	62321	263858	742836
42183	74293	465317	419308

Prove Examples 11 and 16.

SUBTRACTION

Types of Oral Drills for Frequent Practice

4. **1.** Beginning with 100, subtract 6 at each tap of the pencil. Give the remainder.

2. Beginning with 95, subtract 7 until the remainder next below 30 is reached. Write your answer.

3. Beginning with 93, subtract 8 until a remainder ending with the figure 1 is reached. Write your answer.

4. Subtract 9 from each of the following: 41, 62, 35, 47, 76, 93, 84, 68.

5. I had $1.50. I spent 30 ct., 15 ct., 10 ct., 8ct., and 12 ct. How much had I left?

SIGHT EXERCISES
For Oral Drill or for Seat Work in Limited Time

5. Subtract 37 from 55.

HINT. Subtract thus: 55, 25, 18. (Or, 37 and 10 = 47; 47 and 8 = 55. 10 + 8 = 18.)

Find remainders:

1.	100	36	54	38	47	36	85
	34	19	17	29	19	17	31

2.	40	60	90	70	80	50	30
	28	39	56	43	37	26	18

3.	120	150	210	350	260	140	390
	88	63	74	90	85	76	95

4.	136	235	619	561	895	731	586
	114	212	508	343	723	438	427

5.	500	900	600	400	720	818	426
	127	325	436	274	590	309	208

Read each example and give the answer quickly:

6. $59 - 27 = ?$ $84 - 38 = ?$ $67 - 29 = ?$

7. $61 - 23 = ?$ $55 - 29 = ?$ $73 - 36 = ?$

8. 97 ct. $- 28$ ct. $= ?$ $\$1.30 - \$0.40 = ?$

9. 85 ct. $- 69$ ct. $= ?$ $\$1.72 - \$1.50 = ?$

10. $\$2 - \$1.20 = ?$ $\$5 - \$3.60 = ?$

11. $\$10 - \$7.50 = ?$ $\$6 - \$5.28 = ?$

12. 2 hr. $- 85$ min. $= ?$ 1 bu. $- 15$ qt. $= ?$

13. 2 gal. $- 5$ qt. $= ?$ 1 yd. $- 21$ in. $= ?$

WRITTEN EXERCISES

Verify the answers of Examples 2, 5, and 7, of the following examples by adding the remainder to the subtrahend; the sum should equal the minuend.

6. Subtract:

1.	28405	9729	10005	20693
	19641	3984	6666	17358

| 2. | 144000 | 17002 | 417009 | 278000 |
| | 99798 | 16599 | 256674 | 196200 |

| 3. | 2940645 | 357621 | 276003 | 824917 |
| | 1673861 | 160039 | 151297 | 338271 |

| 4. | 257329 | 756127 | 645814 | 582165 |
| | 127841 | 680309 | 294726 | 428973 |

| 5. | 1902470 | 2500400 | 5000000 | 3008760 |
| | 1716839 | 1793157 | 4365128 | 2843162 |

| 6. | $4.01 | $38.65 | $65.00 | $47.50 |
| | 1.39 | 18.75 | 37.63 | 26.84 |

| 7. | $150.00 | $200.00 | $651.80 | $311.18 |
| | 92.67 | 164.50 | 475.00 | 265.78 |

| 8. | $1000.00 | $3040.68 | $2615.49 | $3516.84 |
| | 925.62 | 1950.79 | 1728.51 | 2970.59 |

| 9. | $38400.00 | $46500.00 | $50127.00 |
| | 34925.50 | 39498.95 | 37204.73 |

| 10. | $3647500.00 | $7421936.40 | $526174.28 |
| | 1878250.50 | 5127315.56 | 491590.67 |

| 11. | $1687498.52 | $3247368.12 | $427956.77 |
| | 793709.69 | 2958471.74 | 392789.48 |

MULTIPLICATION

Types of Oral Drills for Frequent Use

7. **1.** Add by 5's to 100.

2. Add by 6's to 12 times 6.

3. Repeat the multiplication table of sevens.

4. Write the eighth multiple of 3; of 6; of 9; of 12; of 5; of 10; of 7; of 4; of 8; of 11.

5. Beginning with 41, add 9 at each tap of the pencil.

6. How much is 11×6? 9×12? 4×8? 7×12?

7. How many are 3 times 5? 6×5? 12×5? 24 times 5?

8. How many are 2×15? 8×15? 16×15?

9. How many are 10 times 23? 5 times 23?

10. $3 \times 7 = 21$; $6 \times 7 = ?$ $9 \times 7 = ?$

11. $(5 \times 8) + (6 \times 8) = ?$ $(3 \times 9) + (12 \times 9) = ?$
$(8 \times 5) - (3 \times 5) = ?$ $(9 \times 8) - (5 \times 8) = ?$

SIGHT EXERCISES

For Rapid Drill From Text or Blackboard; or for Seat Work in Limited Time

8 Read and tell the products:

1. $6 \times 12 = ?$ $7 \times 9 = ?$ $4 \times 7 = ?$ $8 \times 9 = ?$

2. $9 \times 4 = ?$ $12 \times 10 = ?$ $7 \times 6 = ?$ $9 \times 6 = ?$

3. $12 \times 11 = ?$ $9 \times 12 = ?$ $11 \times 8 = ?$ $15 \times 9 = ?$

4. Multiply by 10: 24; 59; 100; 256; 144; 1,728; 3,060.

5. Multiply by 100: 36; 651; 188; 200; 1,756; 7,000; 1,009.

6. Multiply by 1,000: 48; 72; 12; 461; 380.

Read each example and give the answer:

7. 21 multiplied by 6 = ? $21 \times 7 = ?$ $21 \times 8 = ?$

8. $32 \times 4 = ?$ $42 \times 5 = ?$ $61 \times 9 = ?$ $58 \times 2 = ?$

9. $66 \times 3 = ?$ $75 \times 4 = ?$ $81 \times 6 = ?$ $69 \times 9 = ?$

Find products:

10.	28	45	56	87	79	67
	20	30	40	50	60	70

Add the products and tell the sums only:

11. (6×9) and (8×4); (7×9) and (9×3); (8×5) and (11×11)

12. (8×9) and (4×7); (7×7) and (9×4); (12×11) and (8×7)

Read and tell the answer:

13. (6×6) plus (4×9); $(8 \times 8) + (3 \times 6)$; $(9 \times 9) + (7 \times 7)$

14. How much are $(6 \times 5) + (2 \times 5)$? $(5 \times 9) + (4 \times 9)$? $(3 \times 8) + (7 \times 8)$?

15. How much are $(6 \times 5) - (2 \times 5)$? $(5 \times 8) - (2 \times 8)$? $(9 \times 7) - (4 \times 7)$?

16. $(9 \times 5) + (11 \times 4) = ?$ $(7 \times 6) + (4 \times 6) = ?$

$(10 \times 9) - (4 \times 9) = ?$ $(9 \times 6) - (3 \times 6) = ?$

17. $(7 \times 8) - (5 \times 8) = ?$ $(12 \times 6) - (3 \times 6) = ?$

$(12 \times 7) - (5 \times 7) = ?$ $(8 \times 5) - (4 \times 5) = ?$

SIGHT DRILLS

"The Stairs"

9. Tell the products quickly:

$$X \begin{cases} 5,\ 10 \\ 3,\ 6,\ 9 \\ 2,\ 4,\ 8 \\ 6,\ 12 \\ 7,\ 11 \end{cases}$$
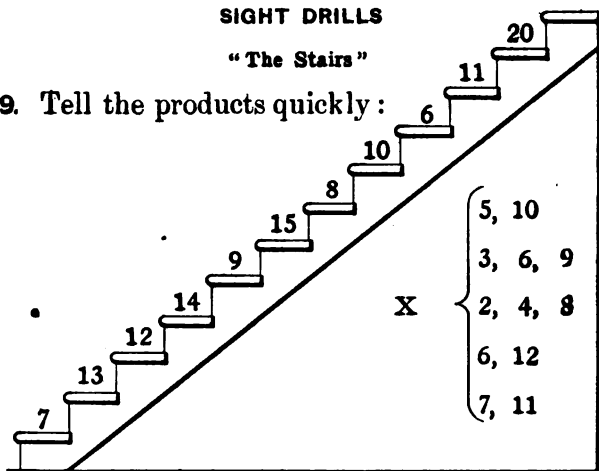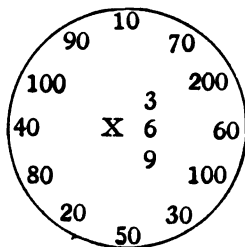
"The Circle"

Multiply each number on the circumference by each number in the center of the circle.

Tell the products only.

9	13	6	12	7	11	15	8
		× 3	5	7	9	11	

7	20	9	14	8	12	11	6	4	15
		× 4	6	8	12				

Begin at the right. Multiply each figure in the upper line by each figure in the lower line. Tell the products quickly.

SIGHT DRILLS

10. Tell products at once:

1. 8×12	**11.** 12×6	**21.** 4×18
2. 7×9	**12.** 8×9	**22.** 5×16
3. 8×7	**13.** 12×9	**23.** 8×13
4. 6×9	**14.** 7×7	**24.** 6×14
5. 7×4	**15.** 6×7	**25.** 12×12
6. 7×8	**16.** 12×4	**26.** 13×12
7. 4×9	**17.** 12×11	**27.** 15×8
8. 4×12	**18.** 6×15	**28.** 7×20
9. 6×4	**19.** 5×14	**29.** 6×25
10. 3×7	**20.** 3×21	**30.** 8×50

31. $2 \times 6 = 12;$ $4 \times 6 = ?$ $6 \times 6 = ?$ $8 \times 6 = ?$

32. $5 \times 2 = 10;$ $10 \times 2 = ?$ $20 \times 2 = ?$ $40 \times 2 = ?$

33. $3 \times 4 = 12;$ $6 \times 4 = ?$ $9 \times 4 = ?$ $12 \times 4 = ?$

34. $4 \times 4 = 16;$ $8 \times 4 = ?$ $12 \times 4 = ?$ $16 \times 4 = ?$

How much are:

35. $(3 \times 4) + (2 \times 4)$?   **38.** $(9 \times 3) - (6 \times 3)$?

36. $(6 \times 5) + (4 \times 5)$?   **39.** $(12 \times 5) - (5 \times 5)$?

37. $(7 \times 2) + (4 \times 2)$?   **40.** $12 \times 5 = ?$; $13 \times 5 = ?$

41. $5 \times 16 = 80$; $5 \times 17 = ?$ $5 \times 18 = ?$

42. $7 \times 20 = 140$; $7 \times 19 = ?$ $7 \times 18 = ?$

43. $144 = 2 \times ?$ $144 = 8 \times ?$ · $144 = 6 \times ?$

44. $120 = 4 \times ?$ $120 = 8 \times ?$ $120 = 12 \times ?$

ORAL PROBLEMS

11. **1.** A man sleeps 8 hr. each night; how many hours will he sleep in one week? in one month (30 da.)?

2. A man sold books at $9 per set; what did he receive for 25 sets?

3. If a train moves at the rate of 12 mi. per hour, what distance will it cover in 12 hr.?

4. There are 6 houses in one row; each house has 15 windows; how many windows in all the houses?

5. If each window has 12 panes of glass, how many panes of glass in all the houses?

6. What is the cost of 25 lb. of beef at 20 ct. per pound?

7. A reader costs $0.50; what will be the cost of all the readers used by this class?

8. A class uses 8 pads of paper in its arithmetic work during one week; how many pads will be required during the entire term of 20 wk.?

9. The class solves 20 problems in mental arithmetic and 25 written problems during the week; at this rate, how- many problems will be solved during the month (counting 20 school days to the month)?

10. What will be the total loss on 12 pieces of cloth, each containing 50 yd., if there is a loss of 10 ct. per yard?

WRITTEN EXERCISES

12. Find products:

1.	286	374	591	870	276	987
	15	25	35	45	55	65

2. Multiply by 24: 1,767; 2,486; 5,671; 8,430.

Find products:

3.	574	807	561	295	749	836	921
	24	42	61	55	18	36	28

4.	395	879	468	527	385	692	729
	136	211	348	139	165	147	134

5.	276	527	836	279	593	647	859
	109	305	407	206	107	603	702

6.

1285	1467	2198	4736	5918	2579
539	426	874	519	279	816

7.

17468	59137	64206	81294	110749
2147	1908	3007	5640	5739

WRITTEN EXERCISES

13. Find products :

1. 239×65
2. 154×87
3. $2,345 \times 290$
4. 756×565
5. $3,214 \times 650$
6. $17,689 \times 6,543$
7. $32,189 \times 34$
8. $6,542 \times 8,900$
9. $34,975 \times 54,300$
10. $127,093 \times 545$

Prove Examples 4, 7, and 10.

WRITTEN PROBLEMS

14. 1. A postman walks 24 mi. every day; what distance will he walk in 30 da.?

2. If it takes a man 55 min. to sew a coat, how long will it take him to sew 25 coats?

3. There are 63 gal. in a hogshead; how many gallons are there in 126 hhd.?

4.* What will be the cost of 234 T. of coal at the rate of $7 per T.?

* NOTE TO TEACHER. An elementary knowledge of the Law of Commutation should be developed because many examples can be

5. At $1.35 per yard, what will a merchant have to pay for 3 rolls of carpet, each roll containing 55 yd.?

6. The cost of building a house is $25,550. What will be the total cost of building 245 similar houses?

7. If 3 cows consume 435 lb. of hay per day, how much will they consume in 23 da. (at that rate)?

8. A clock strikes 180 times a day; how many times will it strike in 67 da.?

9. One acre of land produces 495 lb. of cotton; what will a farm containing 1,245 A. produce?

10. There are 17 cars in a freight train; each car carries 150 bbl. of pork, each barrel of pork weighing 200 lb. What is the weight of the entire freight?

11. The distance between New York and Chicago is approximately 905 mi. How far has a conductor traveled who has made the trip 303 times?

solved easily by the application of this principle; *e.g. A baker bakes 1,826 loaves: each loaf weighs 3 lb. What is the total weight?* Logically, the solution is 1,826 × 3 lb. If, however, the pupils understand that 1,826 × 3 lb. = 3 × 1,826 lb., they may obtain the answer by multiplying 1,826 by 3 instead of multiplying 3 by 1,826.

12. A book contains 148 pages; there are on an average 35 lines to the page, and 11 words to each line. How many words in the book?

13. A regiment of 987 soldiers eats 346 lb. of flour at one meal. Counting two meals during the day, what is the weight of the flour consumed by the regiment in 14 da.?

14. A jeweler buys watch-cases for $ 18.75 each, and watch-movements for $ 12.60 each; and he pays a workman $ 3.30 each for putting movements into cases and adjusting them. How much do 48 complete watches cost him?

15. If advertising costs $ 1.60 per column-inch, how much will an advertisement cost which is 8 in. long and extends across 2 columns.

16. What will be the charge for consuming 2,800 cu. ft. of gas, if the rate is 80 ct. per 1,000 ft.?

17. Make up three problems in multiplication.

DIVISION

15. **1.** Repeat the division table of eights.

2. How much is $72 \div 2$; by 3; by 4; by 6?

3. $124 \div 2 = ?$ $36 \div 3 = ?$ $64 \div 4 = ?$

4. $50 \div 10 = ?$ $100 \div 10 = ?$ $150 \div 10 = ?$

5. One factor of 25 is 5; what is the other factor?

6. Give the sets of factors of 24; 25; 36; 40.

7. How many 6's in 36; in 54; in 60; in 96?

8. $? \div 3 = 12$; $? \div 4 = 12$; $? \div 6 = 12$.

9. $9 \div 3 = 3$; $18 \div 3 = ?$ $27 \div 3 = ?$

10. Add the quotients of $28 \div 4$ and $14 \div 7$.

SIGHT EXERCISES

For Rapid Drill from Text or Blackboard; or for Seat Work in Limited Time

16. Read and tell the quotients at sight:

1. 8)72	7)49	6)72	10)120	9)81
2. 9)36	5)55	5)65	11)132	4)48
3. 11)121	3)63	2)124	12)144	6)66

c   17

4. 9)63   4)144   5)250   6)240   4)1000

5. 3)33   3)99   3)120   3)144   3)939

6. $42 \div 6$ $84 \div 6$ $25 \div 5$ $25 \div 10$ $99 \div 9$

7. $120 \div 5$ $35 \div 7$ $64 \div 8$ $125 \div 5$ $63 \div 3$

8. Divide 100 by 10; by 5; by 20; by 4; by 25; by 40; by 50.

9. Divide 144 by 3, 4, 6, 8, 9, 12, 24.

10. Divide 250 by 5, 10, 20, 25, 50, 100.

Write the answers of the following at the tap of the pencil:

11. 4)1212   5)345   10)12450

12. 8)2448   7)7301   6)1236

13. 3)1236   9)4563 $74214 \div 7$

14. How much are $(144 \div 12) + (120 \div 12)$?

15. What is the difference between $72 \div 9$ and $120 \div 10$?

16. $(96 \div 8) - (56 \div 8) = ?$

17. $(3 \times 4) \div 2 = ?$

18. $(96 \div 5) - (6 \div 5) = ?$

19. Divide the following numbers by 10: 230, 450, 4500, 5600, 14900.

20. Divide the following numbers by 100: 12400, 7600, 234900, 54000.

21. Divide the following numbers by 1000: 23000, 542000, 654000.

22. What is $\frac{1}{4}$ of 3468 ? $\frac{1}{10}$ of 5230 ? $\frac{1}{6}$ of 346 ?

23. $\frac{1}{8}$ of $23400 = $? $\frac{1}{5}$ of $5600 = $? $\frac{1}{7}$ of $4914 = $?

24. Find $\frac{1}{5}$ of 50 ; of 60 ; of 100 ; of 200.

ORAL DRILL

"The Ladder"

17. 1. Begin at the lowest number ; go up as quickly as possible. Tell the quotients.

2. Begin at the highest number ; go down as quickly as possible. Tell the quotients.

3. Begin at the middle and skip about.

4. Make other ladders, writing your own dividends and divisors.

128
80
40
160
32
88
48
104
120
72
96
112
$\div 8$

ORAL DRILL

"Railroad Ties"
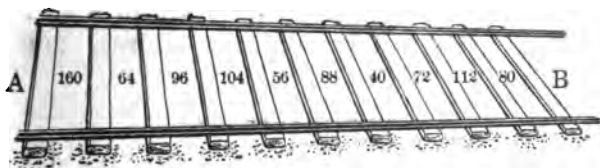
Go from A to B as quickly as possible, then back from B to A.

Divide by 2, by 4, by 8.

18. 1. A boy walks an average of 4 mi. per hour. How long will it take him to walk 52 mi.?

2. Milk is sold at 6 ct. per quart. How many quarts are sold for $1.50?

3. In a classroom there are 48 desks; there are 8 desks in a row; how many rows are there?

4. How much will eggs cost per dozen if $2.40 will purchase 6 doz.?

5. A man receives $3 for working 1 da. of 10 hr. What does he receive per hour?

6. $50 buys 10 T. of coal; how many tons will $10 buy?

7. A boy buys 125 marbles for 25 ct.; how many can he buy for 1 ct.?

8. How many books will $1 buy if $5 will buy 40 books?

9. One foot equals 12 in.; how many feet in 240 in.?

10. A boy addressed 20 envelopes in ½ an hour; how long did it take him to address 200 envelopes?

19. Divide:

1. $2,989 \div 7$		**4.** $47,367 \div 63$
2. $2,672 \div 16$		**5.** $54,000 \div 275$
3. $8,675 \div 32$		**6.** $35,084 \div 49$

7. 72,005 ÷ 305   9. 54,203 ÷ 7001
8.   8,697 ÷ 4789   10. 75,000 ÷ 1250

Prove Examples 3, 6, and 9.

WRITTEN PROBLEMS

20. 1. The President of the United States receives
a salary of $75,000 per year. How much does he
receive per month? per week?   (52 wk. = 1 yr.)

2. How much does he receive per day, counting
365 da. to the year?

3. Five workmen remove 37,575 cu. ft. of earth
in 2 wk. of 6 working days each. How much
does each remove on an average per day?

4. A steamship burns 1,650 T. in 5 da. on a
voyage of 2,750 mi. How much coal is used for
each mile? How much during each day?

5. 126 bbl. of apples were sold for $428.40.
Find the price per barrel.

6. The city of New York gained 1,450,000 in
population during the past ten years. What was
the average increase each year?

7. A bushel of corn weighs 56 lb. How many
bushels are there in 8,766 lb. of corn?

8. A business house loses $7,895 during a year
of 305 working days. What was the average loss
per day? per week?

9. A farmer had a field of 18 A. planted in wheat. The farm yielded 4950 bu. of wheat. Find the average yield per acre.

10. A department store paid $6,732 each week in salaries to its clerks. The salaries averaged $18 per week. How many clerks were employed?

11. Make up three problems in division.

READING EXERCISE

21. By using the signs, *plus* (+) to indicate addition, *minus* (−) to indicate subtraction, the multiplication sign (×), and the division sign (÷), and the parenthesis (( )), all or nearly all of the examples in simple arithmetic may be written in a brief form.

Multiply the sum of 9 and 6 by the difference between 5 and 2.

This example may be written in this way: $(9 + 6) \times (5 - 2)$.

WRITTEN EXERCISES

22. Write the following examples in brief arithmetical form:

1. Multiply the sum of 5 and 9 by the sum of 4 and 3.

2. Multiply the sum of 11 and 9 by the difference between 9 and 4.

3. Divide the sum of 50 and 34 by the sum of 5 and 7.

4. Add the difference between 28 and 12 to the product of 8 and 8.

5. John had 2 quarters, 5 dimes, and 3 nickels. How much money had he?

<div align="center">

SIGHT EXERCISES

</div>

23. In each example the operations within parentheses are to be done first.

Read and give the answer:

1. $(6 + 3) \times (5 + 2) = ?$ $(8 + 9) \times (6 + 4) = ?$

2. $(12 - 7) \times (13 - 4) = ?$ $(18 - 3) \times (17 - 15) = ?$

3. $(9 + 23) \div (5 + 3) = ?$ $(24 \div 4) \times (3 \times 2) = ?$

4. $(80 + 16) \div (13 - 5) = ?$ $(6 \times 5) + (8 + 4) = ?$

5. $(60 + 10) \div (11 - 4) = ?$ $(50 + 6) \div (10 - 2) = ?$

If there are no parentheses, the operations of multiplication and division are to be done first.

6. $9 \times 4 + 6 \times 3 = ?$ $10 \times 5 + 9 \times 6 = ?$

7. $24 \div 4 + 4 \times 3 = ?$ $45 \div 3 - 4 \times 2 = ?$

8. $10 + 8 \times 8 - 2 = ?$ $6 \times 8 + 12 \div 6 = ?$

9. $144 \div 12 + 48 = ?$ $100 \times 34 - 2000 = ?$

10. $7 \times 9 + 7 - 25 = ?$ $150 \times 3 + 50 - 42 = ?$

SIGHT DRILLS

24. Read and give answers :

1. $25 + 45 = ?$
2. $30 + 16 + 14 = ?$
3. $42 - 6 + 18 = ?$
4. $55 - 11 + 26 = ?$
5. $13 + 27 + 50 = ?$
6. $80 - 40 - 30 - 10 = ?$
7. $56 + 14 - 25 = ?$
8. $37 + 16 + 80 = ?$
9. $75 - 30 + 20 + 10 = ?$
10. $100 - 40 - 30 + 26 = ?$
11. $9 \times 15 = ?$
12. $105 + 5 = ?$
13. $8 \times 13 = ?$
14. $160 + 8 = ?$
15. $12 \times 15 = ?$
16. $80 + 16 = ?$
17. $7 \times 16 = ?$
18. $52 + 13 = ?$
19. $5 \times 14 = ?$
20. $45 + 15 = ?$
21. $3 \times 22 = ?$
22. $84 + 21 = ?$
23. $5 \times 25 = ?$
24. $90 + 18 = ?$

25. $(8 \times 50) + (4 \times 50) = ?$
26. $(4 \times 8) + (6 \times 8) = ?$
27. $(5 \times 12) + (7 \times 12) = ?$
28. $(4 \times 15) + (6 \times 15) = ?$
29. $(14 \times 9) - (7 \times 9) = ?$
30. $(21 \times 12) - (9 \times 12) = ?$
31. $6 \times 15 = 90; \ 12 \times 15 = ?$
32. $4 \times 13 = 52; \ 8 \times 13 = ?$
33. $7 \times 14 = 98; \ 14 \times 14 = ?$

34. $8 \times 16 = 128$; $16 \times 16 = ?$

35. $8 \times 12 = 96$; $16 \times 12 = ?$

36. $6 \times 21 = 126$; $12 \times 21 = ?$

37. $45 \times 6 = ?$

38. $248 + 8 = ?$

39. $72 \times 4 = ?$

40. $444 + 6 = ?$

41. $8 \times 36 = ?$

42. $574 + 7 = ?$

43. $6 \times 94 = ?$

44. $837 + 9 = ?$

45. $5 \times 49 = ?$

46. $720 + 12 = ?$

47. $7 \times 34 = ?$

48. $1,620 + 4 = ?$

49. $9 \times 28 = ?$

50. $1,452 + 12 = ?$

ANALYSIS OF PROBLEMS

Explanations or Forms of Analysis in Multiplication and Division

25. 1. A ton of coal weighs 2,000 lb. Find the weight of 26 T.

Explanation or Form of Analysis. 26 T. weigh 26 times as much as 1 T.

Therefore the weight of 26 T. is $26 \times 2,000$ lb. or 52,000 lb. *Ans.*

2. If a carpenter receives $\$22.50$ a week, how much money does he receive in 18 wk.?

Explanation or Form of Analysis. The amount received in 18 wk. is 18 times the amount received in 1 wk.

Therefore the amount received in 18 wk. is $18 \times \$22.50$ or $\$405$ *Ans.*

3. A chair costs $3.80. Find the cost of 96 chairs at that rate.

Explanation or Form of Analysis. The cost of 96 chairs is 96 times the cost of 1 chair.

Therefore the cost of 96 chairs is $96 \times \$3.80$ or $364.80 *Ans.*

4. A storekeeper buys blank books at 40 ct. a dozen and sells them at 76 ct. a dozen. Find his profit on 60 doz. books.

Explanation or Form of Analysis. The profit on 60 doz. books is 60 times the profit on 1 doz. books.

Therefore the profit on 60 doz. is 60×36 ct. $= \$21.60$ *Ans.*

5. 34 bbl. of flour weigh 6,664 lb. Find the weight of 1 bbl. of flour.

Explanation or Form of Analysis. The weight of 1 bbl. is $\frac{1}{34}$ of the weight of 34 bbl.

Therefore the weight of 1 bbl. is $\frac{1}{34}$ of 6664 lb., or $6664 \div 34$, or 196 lb. *Ans.* (Compare with No. 1.)

6. A man receives $450 for working 15 wk. How much does he receive for working 1 wk.?

Explanation or Form of Analysis. The amount received for 1 wk. is $\frac{1}{15}$ of the amount received for 15 wk.

Therefore the amount received for 1 wk. is $\frac{1}{15}$ of $450, or $450 \div 15$ or $30 *Ans.* (Compare with No. 2.)

7. If a man is paid at the rate of $30 a week, how many weeks must he work to earn $480?

Explanation or Form of Analysis. The number of weeks will equal the number of 30's in 480.

Therefore the number of weeks will equal $480 \div 30$ or 16 *Ans.*

8. If 14 tables cost $105.28, how much will 1 table cost?

Explanation or Form of Analysis. The cost of 1 table is $\frac{1}{14}$ of the cost of 14 tables.

Therefore the cost of 1 table is $105.28 ÷ 14 or $7.52 *Ans.* (Compare with No. 3.)

9. 22 blankets cost $143. Find the cost of 7 blankets.

Explanation or Form of Analysis. The cost of 1 blanket is $\frac{1}{22}$ of $143 or $6.50. (See No. 8.)

The cost of 7 blankets is 7 × $ 6.50 or $45.50 *Ans.* (See No. 3.)

10. 24 spoons cost $138. Find the cost of 12 spoons.

Explanation or Form of Analysis. The cost of 12 spoons is $\frac{1}{2}$ the cost of 24 spoons.

Therefore 12 spoons cost $\frac{1}{2}$ of $138 or $ 69 *Ans.*

11. 6 knives cost $27. Find the cost of 30 knives.

Explanation or Form of Analysis. 30 knives cost 5 times as much as 6 knives.

Therefore 30 knives cost 5 × $27 or $135 *Ans.*

NOTE TO TEACHER. These forms of analysis are given as suggestions. Any form that is logical and clear may be accepted. Forms 1, 5, and 7, illustrating respectively the processes of multiplication, partition, and division, are the forms to be memorized if it is desired to memorize any forms of analysis.

MISCELLANEOUS WRITTEN PROBLEMS

26. 1. Mr. Dawson bought 7 oil paintings at the following prices: $65; $137; $86; $49; $250; $175; $52. He paid $175 for frames. Find the total cost of the pictures.

2. The pupils of a school held an entertainment and took in $450. Then they bought 5 pictures at $12 each; 4 pictures at $22 each; a printing press for $60, a set of books for $50, and a statue for $85. How much money was left?

3. (*a*) A theater was open 6 nights. The first night 1,073 people attended; the second night, 941 people; the third, 829; the fourth, 1,169; the fifth, 1,026; the sixth, 1,261. How many people attended the theater during the 6 nights?

3. (*b*) If the persons who attended the theater paid an average of 75 ct. each, how much money was received on the first night? On the third night? On the sixth night?

4. If the theater had only 1,000 seats:

(*a*) On which nights were persons obliged to stand?

(*b*) On which nights were there vacant seats?

5. The cost of publishing 5000 copies of a certain book was $2140. What was the publisher's gain, if all the copies were sold at 90 ct. each?

6. Find the total population of the following cities:

New York, population,		4,776,883
Chicago,	"	2,185,283
Philadelphia,	"	1,549,008
St. Louis,	"	687,029
Boston,	"	670,585
Cleveland,	"	560,683

7. How much greater is the population of New York than the population of Chicago? Than the population of St. Louis?

8. How much greater is the population of Chicago than the combined population of St. Louis and Boston?

9. How much greater is the population of New York than of Chicago and Philadelphia combined?

10. Is the population of New York greater or less than the combined population of the 5 other cities? How much greater or how much less?

11. An owner of a building received $36,600 in rent. He paid $5,700 in taxes, $11,400 for repairs, and $9,700 for other expenses. How much profit was left for the owner?

12. Bought 245 bu. of corn at 42 ct. a bushel and 376 bu. of wheat at $1.03 a bushel. Find the cost of the corn; of the wheat. Find the total cost of the corn and wheat.

13. The boys of a baseball team bought 11 uniforms at $1.75 each; 6 bats at 85 ct. each; 2 gloves for $1.50 each, and 6 baseballs for 50 ct. each. How much money did they spend?

14. A club of 18 men spend $612. What is each member's share of the expense?

15. A train travels 43 mi. an hour for 4 hr. and 48 mi. an hour for 5 hr. What distance does it cover in the 9 hr.?

16. A ship sailed 371 mi. at an average speed of 14 mi. an hour. In how many hours did the ship travel the entire distance?

17. 75 horses were bought for $195 each and sold for $314 each. Find the total profit.

18. The population of New Jersey is 2,537,167. This is 653,498 greater than the population was 10 yr. ago. What was the population 10 yr. ago?

Find the cost in each of the following:

19. 19 wagons at $123 each.

20. 19 sets of harness at $35 each.

21. 245 lb. tea at 47 ct. a pound.

22. 180 lb. coffee at 28 ct. a pound.

23. 96 cans cocoa at 22 ct. a can.

24. 54 lb. chocolate at 38 ct. a pound.

25. 46 gal. maple sirup at $1.10 a gallon.

26. 73 bottles sirup at 16 ct. a bottle.

27. 85 bbl. of flour at $6.75 a barrel.

28. 36 packages of oatmeal at 16 ct. a package.

29. 48 bags of farina at 19 ct. a bag.

30. 27 cans of condensed milk at 15 ct. a can.

31. 35 lb. of hominy at 19 ct. for $3\frac{1}{2}$ lb.

32. 100 cakes of soap at 9 ct. for 2 cakes.

33. 240 cakes of soap at 15 ct. for 6 cakes.

34. 40 lb. black pepper at 10 ct. for $\frac{1}{4}$ lb.

35. 32 lb. ginger at 12 ct. for $\frac{1}{4}$ lb.

36. 28 lb. nutmeg at 12 ct. for $\frac{1}{4}$ lb.

37. 52 cans peaches at $1.75 per dozen cans.

38. 68 cans pears at $2.30 per dozen cans.

39. 72 cans plums at $2.04 per dozen cans.

40. 120 cans apricots at $2.40 per dozen cans.

41. 144 cans tomatoes at 48 ct. for $\frac{1}{2}$ doz. cans.

42. 32 lb. jelly at 30 ct. for a 2-lb. jar.

43. The steamship *Mauretania* made the follow ing daily "runs" expressed in knots in a voyage from New York to Liverpool: 387, 582, 572, 578, 580, and 108.

(*a*) What was the total distance traversed?

(*b*) Find the average speed per hour, the voyage having taken 4 da. and 20 hr.

Find the cost of the quantity mentioned in the third column:

	QUANTITY BOUGHT	TOTAL COST	
44.	18 pairs of skates	$ 27.00	1 pair
45.	29 pairs of shoes	101.50	1 pair
46.	24 shirts	42.00	4 shirts
47.	24 neckties	30.00	6 neckties
48.	15 blankets	63.00	1 blanket
49.	20 pairs of gloves	35.00	100 pairs
50.	17 suits of clothes	408.00	1 suit
51.	60 hats	145.00	12 hats
52.	19 yd. muslin	3.04	1 yd.
53.	40 yd. lace	6.80	8 yd.
54.	18 books	25.20	7 books
55.	18 books	26.10	6 books
56.	70 doz. eggs	29.40	9 doz.
57.	70 doz. eggs	30.24	10 doz.
58.	25 T. of coal	157.50	1 T.
59.	25 T. of coal	183.65	5 T.
60.	4 rugs	149.00	7 rugs
61.	4 rugs	174.35	12 rugs
62.	16 doz. spoons	143.68	9 doz.
63.	16 doz. spoons	187.48	8 doz.
64.	24 doz. forks	263.53	13 doz.
65.	24 doz. forks	278.40	4 doz.
66.	48 doz. knives	623.04	14 doz.
67.	48 doz. knives	650.00	12 doz.
68.	21 lb. sugar	1.08	$3\frac{1}{2}$ lb.

POPULATION OF STATES

	POPULATION IN 1910	POPULATION IN 1900	POPULATION IN 1890
Georgia	2,609,121	2,216,331	1,837,353
Illinois	5,638,591	4,821,550	3,826,352
Indiana	2,700,876	2,516,462	2,192,404
Massachusetts	3,366,416	2,805,346	2,238,947
Michigan	2,810,173	2,420,982	2,093,890
Missouri	3,293,335	3,106,665	2,679,185
New York	9,113,279	7,268,894	5,997,853
Ohio	4,767,121	4,157,545	3,672,329
Pennsylvania	7,665,111	6,302,115	5,258,113
Texas	3,896,542	3,048,710	2,235,527

69. Find the total population in 1910 of the states mentioned. In 1900. In 1890.

70. Arrange the names of the states in order of population. Which state is first? second? etc.

71. How much greater was the population of New York State in 1910 than in 1900? In 1910 than in 1890?

72. How much did Texas gain in population from 1900 to 1910? From 1890 to 1910?

73. How much greater in 1910 was the population of New York than the population of Pennsylvania? Than that of Ohio? Than that of Texas?

D

74. The population of New York is approximately equal to the population of what three other states combined?

75 Make up other problems from the table. Solve the problems.

76. A steamship burns in five days the following quantities of coal: 220 T, 235 T, 213 T, 235 T, 218 T. What was the average consumption per day?

77. Refer to the last example. If the steamship covered 2500 mi. in the 5 da., what was the average consumption of coal per mile?

78. If the coal cost $2.20 per ton, what was the total cost?

PROPERTIES OF WHOLE NUMBERS

FACTORING

27. What two numbers multiplied together produce 6 ? 10 ? 14 ? 15 ? 21 ?

What three numbers multiplied together produce 12 ? 24 ? 20 ?

Factors : Products	Factors : Products	Factors : Products
$2 \times 3 = 6$	$2 \times 7 = 14$	$2 \times 2 \times 2 = 8$
$3 \times 3 = 9$	$3 \times 7 = 21$	$2 \times 3 \times 4 = 24$
$4 \times 3 = 12$	$4 \times 7 = 28$	$4 \times 5 \times 7 = 140$

The **factors of a number** are the whole numbers which, multiplied together, produce the number.

Therefore every product is exactly divisible by each of its factors.

Every exact divisor of a number is a factor of the number. The quotient is the other factor, or the product of other factors if there are more than two.

For example: 2 is a factor of 12, then $12 \div 2$ or 6 is the other factor. Again, 2 and 3 are factors of 12; then $12 \div (2 \times 3)$ or 2 is the third factor of 12.

28. Separating a number into its factors is called **factoring**: *e.g.* $30 = 2 \times 3 \times 5$.

ORAL EXERCISES

Find the missing factor:

1. $2 \times ? = 6$ 2. $4 \times ? = 12$ 3. $7 \times ? = 28$

4. $3 \times ? = 15$ 5. $? \times 6 = 48$ 6. $? \times 12 = 96$

7. Name two factors which produce 4; 6; 10; 14; 15; 16; 18; 20; 21; 22; 24; 25.

8. Name all the factors (or exact divisors) of 10, including 10 and 1; of 15; of 12; of 36; of 20.

9. Give two sets of factors for each of the following numbers:

40	50	70	84
56	64	45	42

PRIME AND COMPOSITE NUMBERS

29. A **prime number** is a number that has no factors except itself and 1; *e.g.* 2, 5, 7, 13, 17, are prime numbers.

A **composite number** is a number that has factors other than itself and 1; *e.g.* 4, 6, 9, 10, 12, are composite numbers.

A **prime factor** is a factor that is a prime number; *e.g.* 7 is a prime factor of 28.

An **even number** is a number that contains 2 as a factor. All numbers that have 2, 4, 6, 8, or 0 in units' place are even numbers.

An **odd number** is a number that does not contain 2 as a factor. All numbers that have 1, 3, 5, 7, or 9 in units' place are odd numbers.

EXERCISES

30. 1. Name all the even numbers from 1 to 10; from 11 to 20; from 31 to 40; from 61 to 70; from 91 to 100.

2. Name all the odd numbers from 1 to 10; from 11 to 20; from 21 to 30; from 41 to 50; from 71 to 80.

3. Name the prime numbers from 1 to 25.

4. Name the composite numbers from 26 to 50.

Tell whether each of the following is prime or composite; if it is composite, name its factors:

5. 1	8	5	26	48	61	75
6. 11	9	13	39	55	63	100

Give the prime factors of the following:

7. 6	9	12	16	20	24	48
8. 8	10	15	18	22	36	80

Tests of Divisibility

ORAL EXERCISE

31. How can you tell whether or not a number is exactly divisible by 10? By 100? By 1000?

How can you tell an even number?

How can you tell a number divisible by 5?

HINT. Give the multiplication table of 5's and notice the digit figure in each product.

How can you tell a number divisible by 3?

HINT. Write a list of numbers from 10 to 60 that are divisible by 3.

How can you tell a number divisible by 9?

HINT. Write a list of numbers from 9 to 99 that are divisible by 9.

Study the following:

TESTS OF DIVISIBILITY

For 10. Is the unit's figure of the number 0?

For 5. Is the unit's figure of the number 0 or 5?

For 2. Is the unit's figure of the number 2, 4, 6, 8, or 0?

For 3. Is the sum of the number's digits divisible by 3?

For 9. Is the sum of the number's digits divisible by 9?

EXERCISES

32. 1. Write ten numbers greater than 100 that are divisible by 2.

2. Write ten other numbers that are divisible by 5.

3. Write ten other numbers that are divisible by 3; write ten that are divisible by 9.

4. Select the numbers that are divisible by 10:

160	684	195	750	990	945
585	5740	906	765	810	662
462	675	280	888	255	360

5. Select from example 4, the numbers divisible by 5.

6. Put into separate columns those divisible by 2, 3, and 9, respectively.

Finding Prime Factors

33. Find the prime factors of 420.

```
5│420      420 ends in 0; divide by 5, which is prime.
2│ 84      84 is even; divide by 2, which is prime.
2│ 42      42 is even; divide by 2.
3│ 21      21 contains 3 and 7, both prime.
   7       420 = 5 × 2 × 2 × 3 × 7.  Hence the prime
        factors are 2, 2, 3, 5, 7.
```

Find the prime factors of these numbers:

1.	12	**2.**	16	**3.**	21	**4.**	24
5.	18	**6.**	30	**7.**	35	**8.**	36
9.	40	**10.**	48	**11.**	42	**12.**	144
13.	120	**14.**	100	**15.**	56	**16.**	64
17.	60	**18.**	72	**19.**	200	**20.**	75
21.	81	**22.**	49	**23.**	80	**24.**	96

GREATEST COMMON DIVISOR

34. What number is a factor of both 4 and 6? Of both 6 and 9?

What number is a divisor of both 8 and 12? Of both 10 and 15?

A **common divisor** or **common factor** of two or more numbers is a number that exactly divides each of the numbers. For example, 5 is a common factor of 15 and 25.

The **greatest common divisor** (**g. c. d.**) of two or more numbers is the greatest number that exactly divides each of them. For example, 6 is the g. c. d. of 12 and 18.

Find the greatest common divisor of 168 and 264.

$$168 = 2 \times 2 \times 2 \times 3 \times 7$$
$$264 = 2 \times 2 \times 2 \times 3 \times 11$$
$$2 \times 2 \times 2 \times 3 = 24, \text{ g. c. d.}$$

RULE. Resolve both numbers into their prime factors. The product of the prime factors that are common to both numbers is their greatest common divisor.

Find the greatest common divisor in each example :

1. 35, 49.	**2.** 25, 50.
3. 42, 48.	**4.** 12, 16, 32.
5. 14, 35.	**6.** 16, 24.
7. 16, 32, 42.	**8.** 32, 40, 48.
9. 48, 64, 120.	**10.** 60, 144.
11. 24, 36, 60.	**12.** 18, 45, 63.
13. 25, 75, 120.	**14.** 56, 63, 77.
15. 72, 108, 144.	**16.** 36, 48, 90.

Cancelation

FOR READING AND DISCUSSION

35. Prove by trial that statements **1, 2, 3,** and **4** are true:

1. $20)\overline{500} = 2)\overline{50}$ **2.** $4)\overline{12} = 2)\overline{6}$

3. $12)\overline{180} = 3)\overline{45}$ **4.** $25)\overline{150} = 5)\overline{30}$

We have the following important

PRINCIPLE. **In any division, both dividend and divisor may be divided by the same number without changing the quotient.**

Statements **1, 2, 3, 4** may be written as follows:

1. $2 \times 10)\overline{50 \times 10} = 2)\overline{50}$ **2.** $2 \times 2)\overline{6 \times 2} = 2)\overline{6}$

3. $3 \times 4)\overline{45 \times 4} = 3)\overline{45}$ **4.** $5 \times 5)\overline{30 \times 5} = 5)\overline{30}$

In **1**, the factor 10 is omitted, or **canceled**, from both dividend and divisor; in **2**, the factor 2 is canceled; in **3**, the factor 4; in **4**, the factor 5.

Cancelation is the process of casting out equal factors from the dividend and divisor.

· ORAL EXERCISES

Give the results of the following examples.

1. $2 \times 5 \overline{)6 \times 5}$ **2.** $4 \times 7 \overline{)12 \times 7}$

3. $5 \times 10 \overline{)15 \times 10}$ **4.** $3 \times 8 \overline{)12 \times 8}$

5. $5 \times 17 \overline{)25 \times 17}$ **6.** $4 \times 21 \overline{)20 \times 21}$

7. $2 \times 13 \overline{)4 \times 13}$ **8.** $3 \times 9 \overline{)18 \times 9}$

9. $7 \times 65 \overline{)14 \times 65}$ **10.** $4 \times 16 \overline{)20 \times 16}$

WRITTEN EXERCISES

36. 1. Divide 10×12 by 6×5.

We write the dividend above the divisor with a line between, thus:

$$\frac{10 \times 12}{6 \times 5}$$

Dividing both dividend and divisor by 6, we have:

$$\frac{10 \times \overset{2}{\cancel{12}}}{\underset{1}{\cancel{6}} \times 5}$$

*Dividing by 5, we have:

$$\frac{\overset{2}{\cancel{10}} \times \overset{2}{\cancel{12}}}{\underset{1}{\cancel{6}} \times \underset{1}{\cancel{5}}}$$

The answer is $\dfrac{2 \times 2}{1 \times 1}$ or **4**.

*NOTE TO TEACHER. *After* the principle of cancelation has been thoroughly assimilated, the subscript 1 need not be written.

Divide, using cancelation:

2. 15×28 by 3×20.   **3.** 15×54 by 5×18.

4. 6×35 by 7×10.   **5.** 14×15 by 8×35.

6. 3×40 by 6×8.   **7.** 18×84 by 14×90.

8. 18×4 by 8×9.   **9.** 72×15 by 30×144.

10. Find the quotient of :

$$12 \times 28 \times 30 \text{ divided by } 20 \times 84.$$

Solve:

11. $(18 \times 30) \div (9 \times 20)$.   **12.** $(7 \times 15) \div (3 \times 10)$.

Solve the following :

13. $\dfrac{4 \times 21 \times 40}{15 \times 32 \times 2} = ?$ **14.** $\dfrac{8 \times 33 \times 45}{2 \times 40 \times 99} = ?$

15. $\dfrac{3 \times 7 \times 9 \times 5}{21 \times 15 \times 30} = ?$ **16.** $\dfrac{5 \times 8 \times 24 \times 6}{40 \times 32 \times 3} = ?$

17. $\dfrac{10 \times 30 \times 48}{32 \times 20 \times 75} = ?$ **18.** $\dfrac{12 \times 21 \times 63}{9 \times 35 \times 60} = ?$

19. $\dfrac{350}{450} = ?$ **20.** $\dfrac{4800}{2000} = ?$

LEAST COMMON MULTIPLE

37. Table of Multiples

Of 2 $\begin{cases} 2 & 4 & 6 & 8 & 10 & 12 \\ 14 & 16 & 18 & 20 & 22 & 24, \text{ etc.} \end{cases}$

Of 3 $\begin{cases} 3 & 6 & 9 & 12 & 15 & 18 \\ 21 & 24 & 27 & 30 & 33 & 36, \text{ etc.} \end{cases}$

Of 4 $\begin{cases} 4 & 8 & 12 & 16 & 20 & 24 \\ 28 & 32 & 36 & 40 & 44 & 48, \text{ etc.} \end{cases}$

Looking at the table, name some numbers that are **multiples** of both 2 and 3.

6, 12, 18, 24, etc, are called **common multiples** of 2 and 3, because they contain both 2 and 3 an exact number of times.

Name some common multiples of 2 and 4; also of 3 and 4; of 2, 3, and 4.

Name other common multiples of 2, 3, and 4.

Name the smallest or least multiple of 2; of 3; of 4.

Name the least common multiple of 2 and 3; of 2 and 4; of 3 and 4; of 2, 3, and 4.

38. The **least common multiple** (l. c. m.) of two or more numbers is the least number that contains each of them an exact number of times.

For example, 12 is the least common multiple of 4 and 6.

Finding the Least Common Multiple

Method of Inspection

EXERCISES

39. Find the l. c. m. of 4, 6, 9, and 12.

The multiples of the largest given number are 12, 24, 36, 48, etc.

Try 12.   12 is a multiple of 4 and 6, but not of 9.

Try 24.   24 is not a multiple of 9.

Try 36.   36 is a multiple of 9; and therefore 36 is the l. c. m. of 4, 6, 9, and 12.

Find the least common multiple:

1. 3, 4, 6.	**7.** 2, 3, 6, 9.
2. 6, 8, 12.	**8.** 8, 12, 16.
3. 4, 5, 8.	**9.** 2, 10, 25.
4. 2, 4, 7.	**10.** 8, 9, 12.
5. 2, 4, 10.	**11.** 4, 5, 12.
6. 2, 3, 8, 12.	**12.** 20, 25, 50.

Name three numbers whose least common multiple is 60; 72; 120; 144; 200.

The Division Method

EXERCISES

40. Find the l. c. m. of 18, 24, 30, 45.

When the given numbers are large, it is usually easier to find the l. c. m. by the "division method," as follows:

```
 2 | 18  24  30  45
 3 |  9  12  15  45
 3 |  3   4   5  15
 5 |  1   4   5   5
   |  1   4   1   1
```

Find all the prime factors by dividing by any prime number that will exactly divide two or more of the given numbers. The product of the prime factors is the l. c. m.

$2 \times 3 \times 3 \times 5 \times 4 = 360$, l. c. m.

Find the least common multiple as in Example 1:

1. 15, 24, 40.
2. 30, 35, 42.
3. 15, 16, 20.
4. 18, 27, 30.
5. 16, 24, 48.
6. 12, 24, 30.
7. 12, 48, 72.
8. 10, 25, 50.
9. 8, 12, 9, 18.
10. 24, 27, 36, 54.
11. 35, 56, 64, 120.
12. 20, 35, 40, 49.

COMMON FRACTIONS

41. Numbers like 1, 6, 15, 100 are called **whole numbers** or **integers**.

Numbers like $\frac{1}{2}$, $\frac{3}{4}$, $\$\frac{5}{8}$, $\frac{2}{3}$ yd. are called **fractions**. These are read: one half, three fourths, five eighths of a dollar, two thirds of a yard, respectively.

A **unit** is one; as 1 apple, $\$1$, 1 ft., 1 doz.

A **fraction** is one or more of the equal parts of a unit.

For example, $\frac{3}{4}$ represents 3 of the 4 equal parts into which the unit is divided.

A fraction has two **terms**, a numerator and a denominator.

For example, 3 and 4 are the terms of the fraction $\frac{3}{4}$.

The **denominator** of a fraction shows into how many equal parts the unit is divided.

It is written below the line. In the fraction $\frac{3}{4}$, the denominator 4 denotes that the unit is divided into 4 equal parts.

The **numerator** of a fraction shows how many of the equal parts are taken to form the fraction.

It is written above the line. In the fraction $\frac{3}{4}$ the numerator 3 denotes how many fourths are taken. Hence $\frac{3}{4}$ means $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ or 3 times $\frac{1}{4}$.

A **proper fraction** is a fraction whose numerator is less than its denominator. For example, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{99}{100}$, are proper fractions.

The value of a proper fraction is less than 1 (unit).

An **improper fraction** is a fraction whose numerator equals or exceeds its denominator. Its value is equal to 1 or is greater than 1.

For example, $\frac{3}{3}$, $\frac{4}{3}$, $\frac{12}{7}$, $\frac{125}{124}$, are improper fractions.

A **fractional unit** is a fraction whose numerator is 1; as $\frac{1}{4}$, $\frac{1}{25}$, $\frac{1}{100}$.

A **mixed number** is a whole number and a proper fraction written together. It represents their sum.

For example, the mixed number $\$5\frac{3}{4}$ represents $\$5 + \$\frac{3}{4}$. It is read "5 dollars and 3 quarters," or "5 and 3 quarter-dollars."

ORAL EXERCISES

42. Read aloud :

1. $\frac{7}{15}$, $\frac{20}{24}$, $\frac{67}{99}$, $\frac{100}{3}$, $116\frac{17}{50}$.

2. $612\frac{3}{8}$ ft., $40\frac{1}{2}$ lb., $90\frac{7}{10}$ in., $\$16\frac{1}{4}$, $18\frac{3}{5}$ mi.

3. $\frac{9}{5}$ ft., $\frac{12}{7}$ yd., $\frac{18}{5}$ lb., $\$\frac{19}{5}$, $\frac{14}{3}$ mi.

4. $315\frac{1}{2}$ da., $207\frac{3}{8}$ gal., $160\frac{1}{5}$ qt., $\$210\frac{1}{2}$, $175\frac{3}{5}$.

E

43. Write in figures:

1. Nineteen eightieths.   **2.** Four fifths.

3. Ten ninths.   **4.** Seven tenths.

5. Seven twelfths.   **6.** One fifteenth.

7 Fifteen sixteenths.   **8.** Nine elevenths.

9. Seven eighths.   **10.** Nineteen thirtieths.

11. One and one half.   **12.** Six and three fifths.

   13. Nine and three quarters.

   14. Ten and three eighths.

   15. Twelve and five twelfths.

   16. Fifteen and one sixth.

   17. Twenty and nine elevenths.

   18. Six and thirteen one-hundredths.

   19. Thirty-three and four sevenths.

   20. One hundred and nine tenths.

   21. Write ten mixed numbers not given above.
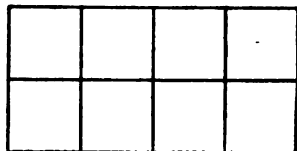
44. Write in words:

1. $\frac{4}{5}$, $\frac{5}{2}$, $18\frac{3}{4}$, $\frac{11}{12}$, $\frac{9}{13}$, $\frac{28}{50}$.

2. $\frac{12}{13}$ in., $66\frac{8}{9}$, $\$14\frac{2}{7}$, $909\frac{9}{10}$, $\frac{67}{17}$.

How Fractions may be Added, Subtracted, Multiplied and Divided like Whole Numbers

45. Similar Fractions are fractions having the same denominator. $\frac{1}{4}$ and $\frac{3}{4}$ are similar fractions. $\frac{1}{4}$ and $\frac{1}{3}$ are *not* similar fractions.

Into how many equal parts is this rectangle divided?

What is each part called?

Looking at the picture, answer the following:

How much is $\frac{1}{8} + \frac{1}{8} + \frac{1}{8}$?

$$\frac{1+1+1}{8} = \frac{3}{8}.$$

How much is $\frac{3}{8} + \frac{2}{8}$? $\frac{3+2}{8} = \frac{5}{8}.$

How much is $\frac{5}{8} - \frac{2}{8}$? $\frac{5-2}{8} = \frac{3}{8}.$

Similar fractions may be added or subtracted by adding or subtracting their numerators, the denominator remaining the same.

How much is $3 \times \frac{1}{8}$? $\frac{3 \times 1}{8} = \frac{3}{8}.$

How much is $2 \times \frac{2}{8}$? $\frac{2 \times 2}{8} = \frac{4}{8}.$

How much is $2 \times \dfrac{3}{8}$? $\dfrac{2 \times 3}{8} = \dfrac{6}{8}$.

A fraction may be multiplied by a whole number by multiplying the numerator by the whole number, the denominator remaining the same.

How much is $\dfrac{6}{8} \div 2$? $\dfrac{6 \div 2}{8} = \dfrac{3}{8}$.

A fraction may be divided by a whole number by dividing the numerator by the whole number, the denominator remaining the same.

SIGHT DRILLS

46. Read and give results rapidly:

1. $\frac{3}{8} + \frac{3}{8} =$	**2.** $\frac{7}{8} - \frac{2}{8} =$	**3.** $\frac{3}{10} + \frac{4}{10} =$
4. $\frac{4}{5} - \frac{1}{5} =$	**5.** $\frac{4}{12} + \frac{5}{12} =$	**6.** $\frac{11}{12} - \frac{5}{12} =$
7. $3 \times \frac{1}{8} =$	**8.** $\frac{4}{8} \div 2 =$	**9.** $4 \times \frac{1}{5} =$
10. $\frac{9}{10} \div 3 =$	**11.** $6 \times \frac{2}{15} =$	**12.** $\frac{12}{15} \div 4 =$
13. $5 \times \frac{3}{20} =$	**14.** $\frac{18}{20} \div 6 =$	**15.** $\frac{3}{10} + \frac{2}{10} =$
16. $\frac{9}{10} - \frac{5}{10} =$	**17.** $7 \times \frac{3}{25} =$	**18.** $\frac{15}{16} \div 5 =$
19. $9 \times \frac{3}{50} =$	**20.** $\frac{27}{100} \div 9 =$	**21.** $\frac{7}{9} - \frac{2}{9} =$

22. Name two similar fractions and add them.

23. Name two similar fractions and subtract them.

24. Make up an example in which a fraction is to be multiplied by a whole number.

25. Make up an example in which a fraction is to be divided by a whole number.

REDUCTION OF FRACTIONS

47. Reduction is the process of changing the form of a number without changing its value.

For example: $2 may be reduced to 200 ct.; 2 lb. to 32 oz.; 9 ft. to 3 yd.; $\frac{1}{2}$ to $\frac{2}{4}$; $\frac{6}{8}$ to $\frac{3}{4}$; $\frac{5}{10}$ to .5; $2\frac{1}{4}$ to $\frac{9}{4}$.

Reducing an Integer or a Mixed Number to a Fraction

Pupils will read each question and give the answer.

1. In $1, how many half dollars are there? How many quarters? How many eighths?

2. How many quarter apples in 2 apples? In 4 apples? In 7 apples? In 10 apples? In 25 apples?

3. How many quarters in $3? In 3\frac{1}{4}$? In 3\frac{1}{2}$? In 3\frac{3}{4}$?

4. How many fifths in 2? In 3$\frac{2}{5}$? In 3$\frac{4}{5}$? In 10$\frac{3}{5}$?

5. Reduce to thirds: 1, 3$\frac{1}{3}$, 4$\frac{2}{3}$, 10, 12$\frac{1}{3}$, 20, 6$\frac{2}{3}$.

6. Are the answers to Question 5 proper or improper fractions?

7. Is a whole number or a mixed number reducible to a proper fraction? Why?

WRITTEN EXERCISES

48. 1. How many fourths in 18?

18

4
___
72   The answer is $\frac{72}{4}$.

We say: There are 4 fourths in 1; in 18 there are 18 times 4 fourths or 72 fourths.

2. How many fourths in $18\frac{3}{4}$?

18^3

4
___
75   The answer is $\frac{75}{4}$.

We say: In 18 units there are 18 times 4 fourths, or $\frac{72}{4}$; and in $18\frac{3}{4}$ there are 72 fourths plus 3 fourths, or $\frac{75}{4}$.

3. How many fourths in 145? In $145\frac{3}{4}$? In $215\frac{1}{4}$? In $76\frac{1}{4}$? In $28\frac{3}{4}$?

Reduce to improper fractions:

4. $62\frac{1}{2}$.   **5.** $102\frac{1}{3}$.   **6.** $66\frac{2}{3}$.   **7.** $114\frac{2}{7}$.   **8.** $55\frac{5}{9}$.

9. $112\frac{5}{6}$.   **10.** $78\frac{7}{12}$.   **11.** $119\frac{3}{7}$.   **12.** $480\frac{3}{25}$.   **13.** $60\frac{7}{30}$.

Change:

14. 25 to tenths.   **15.** 56 to fortieths.

16. 27 to fiftieths.   **17.** $100\frac{1}{3}$ to thirds.

18. $95\frac{17}{25}$ to twenty-fifths.   **19.** $166\frac{4}{9}$ to ninths.

Reduce to improper fractions:

20. $47\frac{5}{18}$.   **21.** $75\frac{9}{32}$.   **22.** $12\frac{71}{100}$.   **23.** $24\frac{26}{35}$.

24. $39\frac{7}{12}$.   **25.** $45\frac{9}{11}$.   **26.** $80\frac{3}{16}$.   **27.** $175\frac{29}{30}$.

28. $\$140\frac{3}{8}$.   **29.** $250\frac{9}{10}$ ft.   **30.** $560\frac{7}{8}$ lb.   **31.** $916\frac{3}{5}$ mi.

32. Tell how to reduce a whole number to a fraction.

33. Tell how to reduce a mixed number to a fraction.

34. Make up ten examples in the reduction of whole numbers or mixed numbers to fractions.

Reducing an Improper Fraction to a Whole Number or to a Mixed Number

ORAL EXERCISE

49. 1. How many dollars in 4 half dollars? In 8 half dollars? In 10 half dollars? In 20 half dollars?

2. How many whole quarts in 7 half quarts? In $\frac{11}{2}$ qt.? In $\frac{25}{4}$ qt.?

3. How many wholes in $\frac{10}{2}$? In $\frac{15}{2}$?

4. How many units may be formed from $\frac{12}{4}$? From $\frac{20}{4}$? From $\frac{21}{4}$?

5. Express as whole numbers: $\frac{9}{3}$, $\frac{24}{3}$, $\frac{18}{3}$, $\frac{12}{6}$, $\frac{30}{10}$, $\frac{24}{4}$, $\frac{35}{7}$.

6. Express as mixed numbers: $\frac{7}{3}$, $\frac{11}{3}$, $\frac{16}{5}$, $\frac{17}{5}$, $\frac{18}{5}$, $\frac{15}{4}$, $\frac{25}{6}$.

WRITTEN EXERCISES

50. Reduce $\frac{75}{4}$ to a whole number or to a mixed number:

$\frac{75}{4} = 75 \div 4$ **Explanation.** $\frac{4}{4} = 1$; $\frac{75}{4} =$ as many 1's as
   $= 18\frac{3}{4}$. $75 \div 4 = 18\frac{3}{4}$ *Ans.*

PRINCIPLE. **A fraction is an indicated division with the dividend above the line and the divisor below it.**

Reduce to whole numbers or to mixed numbers:

1. $\frac{25}{5}$ 2. $\frac{72}{7}$ 3. $\frac{95}{3}$ 4. $\frac{126}{2}$

5. $\frac{134}{9}$ 6. $\frac{275}{13}$ 7. $\frac{650}{15}$ 8. $\frac{229}{4}$

9. $\frac{761}{18}$ 10. $\frac{428}{24}$ 11. $\frac{365}{30}$ 12. $\frac{500}{65}$

13. $\frac{349}{35}$ 14. $\frac{1728}{12}$ 15. $\frac{5760}{16}$ 16. $\frac{2900}{38}$

ORAL PROBLEMS

51. 1. A ticket agent has 190 half dollars. How many dollars has he?

2. A grocer has 38 bags of sugar; each bag contains $\frac{1}{2}$ lb. How many pounds of sugar?

3. A girl has 17 pieces of ribbon. Each piece is $\frac{1}{4}$ yd. long. How many yards in all the pieces?

4. A track is $\frac{1}{8}$ mi. long. A man rides around the track 20 times. How many miles does he ride?

5. A milkman sold 19 bottles of cream. Each bottle contained $\frac{1}{2}$ pt. How many pints of cream were sold?

6. The milkman sold also 39 bottles of milk, each bottle containing $\frac{1}{2}$ qt. How many quarts of milk were sold?

7. A carpenter used 11 pieces of wood, each $\frac{1}{4}$ ft. long. How many feet of wood did he use?
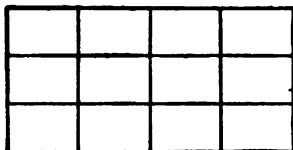
8. In making a wire cage a man needed 43 pieces of wire, each $\frac{1}{4}$ of a yard long. How many yards of wire were used?

Two Important Principles of Fractions

First Principle

READING EXERCISE

52. Looking at the picture, tell how many twelfths in $\frac{1}{2}$? In $\frac{1}{3}$? In $\frac{1}{4}$? In $\frac{2}{3}$? In $\frac{3}{4}$? In $\frac{5}{6}$?

Writing these results in a table, we have:

A. $\frac{1}{2} = \frac{6}{12}$.   *C.* $\frac{1}{4} = \frac{3}{12}$.   *E.* $\frac{3}{4} = \frac{9}{12}$.

B. $\frac{1}{3} = \frac{4}{12}$.   *D.* $\frac{2}{3} = \frac{8}{12}$.   *F.* $\frac{5}{6} = \frac{10}{12}$.

We might have obtained $\frac{1}{2} = \frac{6}{12}$ (without looking at the picture) by multiplying both the numerator and the denominator of $\frac{1}{2}$ by 6, thus

$$\frac{1}{2} = \frac{6 \times 1}{6 \times 2} = \frac{6}{12}.$$

Likewise we have:

B. $\frac{1}{3} = \frac{4 \times 1}{4 \times 3} = \frac{4}{12}$.   *D.* $\frac{2}{3} = \frac{4 \times 2}{4 \times 3} = \frac{8}{12}$.

C. $\frac{1}{4} = \frac{3 \times 1}{3 \times 4} = \frac{3}{12}$.   *F.* $\frac{5}{6} = \frac{2 \times 5}{2 \times 6} = \frac{10}{12}$.

This operation is called the **reduction of fractions to higher terms.**

PRINCIPLE.   **Multiplying both terms of a fraction by the same number does not change the value of the fraction.**

Reduction to Higher Terms

ORAL EXERCISES

53. 1. Reduce $\frac{5}{6}$ to twelfths.

$\frac{5}{6} = \frac{10}{12}$ *Ans.*

Explanation. To make the denominator 12, we must multiply 6 by 2; multiplying both terms by 2, we have $\frac{2 \times 5}{2 \times 6} = \frac{10}{12}$ *Ans.*

WRITTEN EXERCISES

1. Reduce to tenths: $\frac{1}{2}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$.

2. Reduce to sixteenths: $\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{8}{8}$.

3. Reduce to twenty-fourths: $\frac{5}{6}, \frac{3}{8}, \frac{5}{12}, \frac{2}{3}, \frac{7}{8}, \frac{1}{2}, \frac{3}{4}$.

4. Reduce to thirty-sixths: $\frac{5}{9}, \frac{2}{3}, \frac{3}{4}, \frac{7}{12}, \frac{11}{18}, \frac{1}{6}, \frac{2}{9}$.

5. Reduce to sixtieths: $\frac{3}{5}, \frac{5}{12}, \frac{2}{3}, \frac{7}{30}, \frac{5}{6}, \frac{9}{10}, \frac{1}{4}, \frac{4}{15}$.

6. Reduce to hundredths: $\frac{3}{5}, \frac{3}{4}, \frac{1}{2}, \frac{4}{5}, \frac{7}{10}, \frac{7}{20}, \frac{11}{25}$.

7. Reduce to fractions having the denominator 1000: $\frac{3}{8}, \frac{1}{8}, \frac{5}{8}, \frac{7}{8}, \frac{19}{40}$.

ORAL EXERCISES

54. Read and give answers rapidly:

1. $\frac{1}{4} = \frac{}{8} = \frac{}{12} = \frac{}{20} = \frac{}{24} = \frac{}{40}$.

2. $\frac{3}{7} = \frac{}{21} = \frac{}{14} = \frac{}{35} = \frac{}{70} = \frac{}{140}$.

3. Change: $\frac{1}{2} = \frac{}{16}$ **4.** $\frac{3}{4} = \frac{}{12}$

$\frac{3}{4} = \frac{}{16}$ $\frac{2}{3} = \frac{}{12}$

$\frac{7}{8} = \frac{}{16}$ $\frac{5}{6} = \frac{}{12}$

5. Tell how to reduce a fraction to higher terms.

6. Give five new examples in the reduction of fractions to higher terms.

Reduce to higher terms:

7. $\frac{3}{8} = \frac{}{24}$ 8. $\frac{5}{6} = \frac{}{18}$ 9. $\frac{2}{3} = \frac{}{36}$

10. $\frac{9}{10} = \frac{}{50}$ 11. $\frac{4}{5} = \frac{}{35}$ 12. $\frac{7}{8} = \frac{}{32}$

13. $\frac{5}{9} = \frac{}{36}$ 14. $\frac{11}{12} = \frac{}{60}$ 15. $\frac{3}{4} = \frac{}{90}$

16. $\frac{1}{2} = \frac{}{72}$ 17. $\frac{3}{8} = \frac{}{24}$ 18. $\frac{2}{3} = \frac{}{12}$

$\frac{1}{3} = \frac{}{12}$ $\frac{1}{4} = \frac{}{24}$ $\frac{5}{6} = \frac{}{12}$

$\frac{1}{4} = \frac{}{12}$ $\frac{2}{3} = \frac{}{24}$ $\frac{1}{2} = \frac{}{12}$

19. $\frac{7}{8} = \text{---}$ 20. $\frac{2}{3} = \text{---}$ 21. $\frac{7}{9} = \text{---}$

$\frac{5}{6} = \text{---}$ $\frac{7}{12} = \text{---}$ $\frac{5}{6} = \text{---}$

22. $14\frac{1}{4} = \frac{}{8}$ 23. $\frac{85}{21} = \text{---}$ 24. $20\frac{1}{3} = \frac{}{15}$

55.   **Reduction to Lower Terms**

$\frac{1}{2} = \frac{4}{8}$

$\frac{4}{8} = \frac{1}{2}$

$\frac{1}{2} = \frac{4 \times 1}{4 \times 2} = \frac{4}{8}$

$\frac{4}{8} = \frac{8 \div 4}{4 \div 4} = \frac{1}{2}$

When we change $\frac{1}{2}$ to $\frac{4}{8}$, we reduce $\frac{1}{2}$ to higher terms.

When we change $\frac{4}{8}$ to $\frac{1}{2}$, we reduce $\frac{4}{8}$ to lower terms.

To reduce $\frac{1}{2}$ to higher terms, we multiply the numerator and the denominator by the same number.

To reduce $\frac{4}{8}$ to lower terms, we divide the numerator and the denominator by 4.

SECOND PRINCIPLE. **Dividing both terms of a fraction by the same number does not change the value of the fraction.**

Canceling. Dividing both terms of a fraction by the same divisor or factor is called canceling the common divisor or common factor. (See Art. 35.)

A fraction is in its **lowest terms** when the terms of the fraction contain no common divisor.

RULE. **To reduce a fraction to lowest terms, cancel all factors common to both numerator and denominator. (Or, divide the numerator and the denominator by their greatest common divisor.)**

SIGHT EXERCISES

1. Reduce $\frac{18}{24}$ to lowest terms.

$$\frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9}{12}$$

Explanation. Dividing both terms by 2, we get $\frac{9}{12}$.

$$\frac{9}{12} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4} \quad Ans.$$

Dividing both terms by 3, we get $\frac{3}{4}$.

Or

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4} \quad Ans.$$

Or, if we know the greatest common divisor of both terms, we may obtain the answer by one reduction.

ORAL EXERCISES

56. Reduce to lowest terms:

1. $\frac{8}{12}$	**2.** $\frac{15}{20}$	**3.** $\frac{16}{18}$	**4.** $\frac{21}{24}$	**5.** $\frac{12}{18}$
6. $\frac{15}{25}$	**7.** $\frac{12}{24}$	**8.** $\frac{6}{18}$	**9.** $\frac{9}{36}$	**10.** $\frac{5}{25}$
11. $\frac{20}{30}$	**12.** $\frac{14}{56}$	**13.** $\frac{30}{48}$	**14.** $\frac{28}{56}$	**15.** $\frac{36}{81}$
16. $\frac{32}{48}$	**17.** $\frac{75}{125}$	**18.** $\frac{96}{108}$	**19.** $\frac{240}{900}$	**20.** $\frac{72}{240}$

WRITTEN EXERCISES

57. Reduce to lowest terms:

1. $\frac{56}{140}$ 2. $\frac{30}{120}$ 3. $\frac{42}{150}$ 4. $\frac{60}{210}$ 5. $\frac{144}{288}$

6. $\frac{112}{144}$ 7. $\frac{120}{135}$ 8. $\frac{49}{280}$ 9. $\frac{70}{112}$ 10. $\frac{105}{126}$

11. $\frac{84}{112}$ 12. $\frac{72}{200}$ 13. $\frac{21}{189}$ 14. $\frac{225}{360}$ 15. $\frac{75}{120}$

16. $\frac{176}{192}$ 17. $\frac{99}{135}$ 18. $\frac{108}{156}$ 19. $\frac{160}{248}$ 20. $\frac{350}{650}$

21. $\frac{375}{600}$ 22. $\frac{210}{385}$ 23. $\frac{144}{360}$ 24. $\frac{154}{231}$ 25. $\frac{380}{480}$

26. Test your results by reducing your answers to higher terms with the original denominators.

Least Common Denominator

58. Fractions which have the same denominator are said to have a common denominator, and are called **similar fractions**.

For example, $\frac{3}{8}$ and $\frac{7}{8}$ are similar fractions. Their common denominator is 8.

The **least common denominator** (l. c. d.) of two or more fractions is the smallest common denominator to which they can be reduced; it is the least common multiple of their denominators.

Two methods of finding the least common multiple of two or more numbers are described in Arts. 39 and 40. For all, or nearly all, of the examples in this book the l. c. d. can be found by the method of inspection.

1. Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{5}{8}$ **to the least common denominator.**

$\frac{2}{3} = \frac{16}{24}$

$\frac{5}{6} = \frac{20}{24}$

$\frac{3}{4} = \frac{18}{24}$

$\frac{5}{8} = \frac{15}{24}$

Explanation. To find the l.c.d. try the multiples of the largest denominator.

These are 8, 16, 24, 32, etc.

8 is a multiple of 4, but not of 3 or 6.

16 is a multiple of 4, but not of 3 or 6.

24 is a multiple of 4, 3, and 6.

24 is the l.c.d.

Reduce each fraction to twenty-fourths.

ORAL EXERCISES

59. Reduce to the least common denominator:

1. $\frac{2}{3}$, $\frac{1}{2}$. **2.** $\frac{3}{4}$, $\frac{5}{6}$, $\frac{1}{2}$. **3.** $\frac{7}{10}$, $\frac{1}{2}$. **4.** $\frac{2}{3}$, $\frac{4}{5}$.

5. $\frac{5}{8}$, $\frac{7}{12}$. **6.** $\frac{4}{9}$, $\frac{5}{6}$, $\frac{11}{12}$. **7.** $\frac{5}{18}$, $\frac{7}{12}$. **8.** $\frac{4}{5}$, $\frac{9}{10}$, $\frac{1}{2}$.

9. $\frac{5}{11}$, $\frac{2}{3}$. **10.** $\frac{7}{9}$, $\frac{1}{6}$, $\frac{1}{3}$. **11.** $\frac{3}{7}$, $\frac{1}{10}$, $\frac{4}{5}$. **12.** $\frac{7}{12}$, $\frac{5}{9}$, $\frac{1}{6}$.

WRITTEN EXERCISES

60. Reduce to similar fractions:

1. $\frac{5}{12}$, $\frac{7}{15}$, $\frac{3}{4}$. **2.** $\frac{8}{9}$, $\frac{2}{3}$, $\frac{11}{12}$, $\frac{25}{36}$. **3.** $\frac{4}{25}$, $\frac{3}{4}$, $\frac{7}{10}$, $\frac{1}{2}$, $\frac{2}{5}$.

4. $\frac{12}{35}$, $\frac{5}{7}$, $\frac{4}{5}$, $\frac{3}{10}$. **5.** $\frac{1}{8}$, $\frac{5}{16}$, $\frac{17}{24}$, $\frac{1}{6}$. **6.** $\frac{5}{12}$, $\frac{7}{9}$, $\frac{7}{18}$, $\frac{2}{3}$.

For the following examples the *Method of Division*, Art. 40, may be used.

Reduce to fractions having the least common denominator:

7. $\frac{4}{25}$, $\frac{7}{40}$. **8.** $\frac{62}{125}$, $\frac{2}{3}$, $\frac{14}{25}$. **9.** $\frac{17}{30}$, $\frac{59}{105}$, $\frac{25}{42}$.

10. $\frac{11}{18}$, $\frac{47}{90}$, $\frac{31}{120}$. **11.** $\frac{9}{200}$, $\frac{13}{50}$, $\frac{7}{75}$. **12.** $\frac{7}{28}$, $\frac{11}{30}$, $\frac{5}{7}$.

ADDITION OF FRACTIONS

61. Before adding or subtracting fractions, we must reduce them to similar fractions; *i.e.* to fractions having the same denominator. It is best to reduce them to their least common denominator.

1. Add $\frac{5}{12}$, $\frac{7}{10}$, $\frac{3}{4}$.

Process

$$\frac{5}{12} = \frac{25}{60}$$
$$\frac{7}{10} = \frac{42}{60}$$
$$\frac{3}{4} = \frac{45}{60}$$
$$\frac{112}{60} = 1\frac{52}{60} = 1\frac{13}{15}$$
Ans.

Explanation. To find the l.c.d., try 12, 24, 36, 48, 60. $\frac{5}{12} = \frac{25}{60}$; $\frac{7}{10} = \frac{42}{60}$; $\frac{3}{4} = \frac{45}{60}$.

Adding the numerators, we have

$$\frac{25 + 42 + 45}{60} = \frac{112}{60} = 1\frac{52}{60} = 1\frac{13}{15}$$
Ans.

WRITTEN EXERCISES

62. Find sums:

2. $\frac{8}{9} + \frac{11}{12} + \frac{17}{24}$.

3. $\frac{23}{24} + \frac{7}{8} + \frac{9}{16}$.

4. $\frac{7}{8} + \frac{1}{12} + \frac{5}{6} + \frac{2}{3}$.

Add:

5. $\frac{1}{3}$, $\frac{5}{6}$, $\frac{7}{10}$, $\frac{2}{5}$.

6. $\frac{4}{9}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{12}$.

7. $\frac{5}{9}$, $\frac{3}{5}$, $\frac{2}{3}$.

64 COMMON FRACTIONS

WRITTEN EXERCISES

63. 1. Add $25\frac{5}{8}$, $109\frac{5}{6}$, $70\frac{1}{3}$.

You may arrange your work in this way:

$$20\frac{5}{8} = 20\frac{15}{24}$$
$$109\frac{5}{6} = 109\frac{20}{24}$$
$$70\frac{1}{3} = 70\frac{8}{24}$$
$$199\frac{43}{24} = 200\frac{19}{24}$$

Or you may arrange your work in this way:

$$\begin{array}{r|l} & 24 \\ 20\frac{5}{8} & \overline{15} \\ 109\frac{5}{6} & 20 \\ 70\frac{1}{3} & 8 \\ \hline 199 & \frac{43}{24} = 200\frac{19}{24} \end{array}$$

2. Add $24\frac{1}{2}$, $16\frac{2}{3}$, $8\frac{1}{4}$.

3. Add $50\frac{5}{8}$, $36\frac{3}{4}$, $24\frac{2}{5}$.

4. Add $73\frac{5}{8}$, $19\frac{1}{6}$, $24\frac{5}{12}$.

5. Add $26\frac{1}{2}$, $40\frac{3}{5}$, $29\frac{9}{10}$.

6. Add $48\frac{2}{3}$, $61\frac{7}{9}$, $38\frac{1}{12}$.

7. Add $34\frac{1}{5}$, $27\frac{5}{8}$, $48\frac{7}{10}$.

8. Add $125\frac{5}{9}$
$260\frac{3}{4}$
$185\frac{1}{6}$

9. Add $148\frac{2}{7}$
$361\frac{1}{3}$
$403\frac{5}{9}$

10. Add $84\frac{7}{8}$
$69\frac{3}{5}$
$47\frac{3}{4}$

11. Add $106\frac{2}{3}$
$93\frac{3}{8}$
$64\frac{5}{6}$

12. Add $47\frac{5}{12}$
$19\frac{3}{8}$
$24\frac{5}{9}$

13. Add $305\frac{1}{2}$
$271\frac{7}{9}$
$168\frac{3}{8}$

SUBTRACTION OF FRACTIONS

64. 1. Subtract $12\frac{3}{4}$ from $18\frac{5}{6}$.

You may arrange your work according to either model:

$$18\frac{5}{6} = 18\frac{10}{12}$$
$$12\frac{3}{4} = 12\frac{9}{12}$$
$$\overline{\qquad 6\frac{1}{12}}\ Ans.$$

$$\begin{array}{c} 12 \\ 18\frac{5}{6}\ \boxed{10} \\ 12\frac{3}{4}\ \ 9 \\ \overline{6\ \ \frac{1}{12}}\ Ans. \end{array}$$

65. 2. Subtract $12\frac{3}{4}$ from $18\frac{1}{6}$.

You may arrange your work according to either model:

$$18\frac{1}{6} = 18\frac{2}{12}$$
$$12\frac{3}{4} = 12\frac{9}{12}$$
$$\overline{\qquad 5\frac{5}{12}}\ Ans.$$

$$\begin{array}{c} 12 \\ 18\frac{1}{6}\ \boxed{2} \\ 12\frac{3}{4}\ \ 9 \\ \overline{5\ \ \frac{5}{12}}\ Ans. \end{array}$$

Explanations. We cannot take $\frac{9}{12}$ from $\frac{2}{12}$. We may, however, do one of two things:

1. Change one of the units in the minuend to $\frac{12}{12}$, add to it the $\frac{2}{12}$, making the minuend $17\frac{14}{12}$; subtract $12\frac{9}{12}$ from $17\frac{14}{12}$, getting $5\frac{5}{12}$ for the answer.

Or 2. Change one of the units in the minuend to $\frac{12}{12}$; subtract $\frac{9}{12}$ from $\frac{12}{12}$, getting $\frac{3}{12}$; add $\frac{3}{12}$ and $\frac{2}{12}$, getting $\frac{5}{12}$; subtract 12 from 17. Answer $5\frac{5}{12}$. Or, we may say: 9 from 12 leaves 3; 3 and 2 are 5; write down $\frac{5}{12}$; 12 from 17 leaves 5; write down 5.

Note. The Austrian method of subtracting whole numbers may be used in finding the difference between the numerators of similar fractions. (See Book I, p. 33.)

66. 1. $\frac{1}{4}+\frac{1}{8}+\frac{1}{2}-\frac{3}{8}=?$ 2. $\frac{1}{3}+\frac{5}{6}-\frac{1}{2}=?$

3. $\frac{5}{6}-\frac{1}{3}+\frac{1}{2}+\frac{1}{6}=?$ 4. $\frac{2}{3}+\frac{3}{4}-\frac{1}{2}-\frac{1}{12}=?$

5. $\frac{3}{8}+\frac{7}{10}-\frac{1}{2}=?$ 6. $\frac{7}{8}+\frac{3}{4}-\frac{3}{8}+\frac{1}{2}=?$

WRITTEN EXERCISES

67. Perform the operations indicated:

1. $200\frac{1}{2}-150\frac{5}{6}$ 2. $35\frac{1}{6}-17\frac{3}{5}$

3. $46\frac{7}{8}+15\frac{4}{5}+3\frac{1}{3}$ 4. $80\frac{1}{3}+56\frac{2}{7}$

5. $\frac{9}{20}+14+\frac{2}{3}$ 6. $24\frac{1}{5}+13\frac{7}{8}+12\frac{3}{4}$

7. $\frac{15}{28}+7\frac{4}{7}+9\frac{3}{4}$ 8. $13\frac{1}{6}+20\frac{5}{8}-12\frac{1}{2}$

9. $10\frac{5}{12}+9\frac{5}{6}+\frac{3}{4}-5\frac{1}{2}$ 10. $20\frac{3}{8}+9\frac{1}{12}+16\frac{3}{5}$

11. $35\frac{11}{12}+16\frac{7}{18}+11\frac{13}{24}$ 12. $25\frac{3}{8}+17\frac{5}{16}+1\frac{13}{24}$

13. $26\frac{1}{3}+80\frac{5}{9}-15\frac{1}{6}$ 14. $40\frac{1}{5}+26\frac{19}{10}-7\frac{3}{20}$

15. $38\frac{4}{5}+27\frac{2}{3}-20\frac{5}{6}$ 16. $21\frac{5}{6}+13\frac{7}{12}-14\frac{7}{9}$

17. $27\frac{1}{5}-9\frac{5}{8}-2\frac{1}{6}$ 18. $18\frac{1}{4}+15\frac{7}{12}+16\frac{5}{6}$

19. $30\frac{1}{4}+27\frac{5}{12}+6\frac{5}{9}-8\frac{2}{3}$ 20. $100\frac{5}{7}+53\frac{7}{9}+27\frac{16}{63}$

WRITTEN PROBLEMS

68. 1. William earned $\$12\frac{1}{4}$ in one month and $\$18\frac{3}{4}$ in a second month. How much did he earn in both months?

2. George had $\$17\frac{3}{4}$ and spent $\$12\frac{1}{2}$. How much has he now?

3. A piece of lace $3\frac{7}{8}$ yd. long was cut from a piece $9\frac{1}{2}$ yd. long. How many yards were left?

4. A boy is carrying 3 packages. The first weighs $4\frac{1}{8}$ lb.; the second, $5\frac{3}{4}$ lb.; the third, $9\frac{3}{8}$ lb. Find the total weight of the packages.

5. One boy weighs $92\frac{3}{4}$ lb.; another, $87\frac{1}{2}$ lb. How much do both weigh?

6. A butcher buys $97\frac{1}{2}$ lb. of meat. He sells 16 lb. to one customer, $18\frac{3}{4}$ lb. to another, and $21\frac{1}{2}$ lb. to a third customer. (*a*) Find the number of pounds sold. (*b*) Find the number of pounds not sold.

7. (*a*) Find the total length of two boards, if one is $9\frac{1}{4}$ ft. long, and the other $12\frac{1}{8}$ ft. long.

(*b*) How much longer is one board than the other?

8. A boy whose pay was $ $3\frac{3}{4}$ a week received an increase of $ $1\frac{1}{2}$. How much does he receive now?

9. A book weighing $4\frac{3}{4}$ lb. is packed in a wooden box. The box weighs $\frac{7}{8}$ lb. Find the total weight of the book and the box.

10. A boy walks $\frac{5}{6}$ of a mile to school, $\frac{2}{3}$ of a mile to visit a friend, and $\frac{1}{2}$ a mile to reach home. How far has he walked?

11. The distance from A to B is $11\frac{3}{8}$ mi.; from B to C, $10\frac{5}{8}$ mi.; from C to D $15\frac{1}{4}$ mi. Find the distance from B to D. From A to D.

$11\frac{3}{8}$ mi.	$10\frac{5}{8}$ mi.	$15\frac{1}{4}$ mi.
A	B	C	D
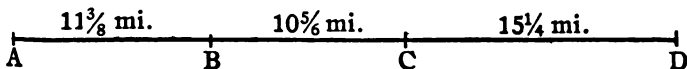
12. A grocer sold $18\frac{3}{4}$ lb. of flour, $16\frac{1}{2}$ lb., 15 lb., and $12\frac{1}{2}$ lb. How many pounds were sold?

13. How many pounds were left if the grocer had 90 lb. at first?

14. Find the total weight of three baskets of wheat if the first basket weighs $15\frac{7}{8}$ lb., the second, $13\frac{3}{4}$ lb., and the third, $19\frac{1}{2}$ lb.

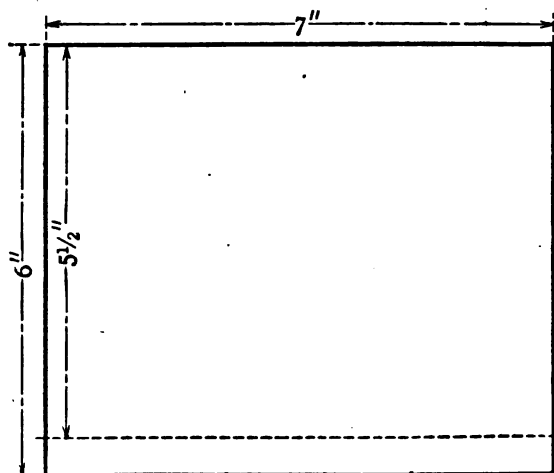**15.** Two boards are $4\frac{5}{8}$ in. and $7\frac{1}{4}$ in. wide.

(*a*) Find their total width.

(*b*) Are they wide enough to make the cover of a box 12 in. wide? Why?

16. A pane of glass $5\frac{1}{2}$ in. wide and 7 in. long is cut from a pane 6 in. wide and 7 in. long.

(*a*) What is the length of the piece left over?

(*b*) What is the width of the piece left over?

17. A carpenter wants to make a box cover $12\frac{1}{8}$ in. wide. He has two boards, one $5\frac{1}{2}$ in. wide and the other $7\frac{7}{8}$ in. wide. How much must be cut from the second board to get the proper width?

18. A pane of glass $10\frac{7}{8}$ in. wide and 15 in. long is cut from a larger piece which is 12 in. wide and 15 in. long.

(a) Find the length of the piece left over.

(b) Find the width of the piece left over.

19. A book which was marked $\$1\frac{1}{2}$ was reduced to $\$\frac{9}{10}$. How much was it reduced?

20. In making a picture frame, a carpenter uses four pieces of molding. Their lengths are; $15\frac{3}{4}$ in., $8\frac{1}{2}$ in., $15\frac{3}{4}$ in., and $8\frac{1}{2}$ in.

(a) Draw a diagram of the picture frame.

(b) Find the total length of the four pieces.

21. In making a desk 13 in. wide a carpenter uses two boards, one $4\frac{3}{8}$ in. wide and the other $9\frac{1}{2}$ in. wide. How wide a strip must be cut from one of the boards to get the proper width?

22. One boy throws a hammer $18\frac{1}{4}$ ft.; another throws it $13\frac{7}{12}$ ft. How much farther does the first boy throw than the second?

23. A tub of butter contained 56 lb. The following amounts were sold; $9\frac{3}{4}$ lb., $7\frac{1}{2}$ lb., 5 lb., $16\frac{3}{4}$ lb. How many pounds were left?

24. A boy spent $\frac{3}{4}$ of an hour studying arithmetic, $\frac{5}{6}$ of an hour studying geography, and $\frac{1}{2}$ hr. making a drawing.

(*a*) How much time did he spend at all his work?

(*b*) How much more time was spent at geography than at arithmetic?

25. A dealer wants to buy 150 bu. of apples. He obtains $30\frac{1}{2}$ bu. from one farmer, 75 bu. from another, and $25\frac{3}{4}$ bu. from a third. How many bushels are still needed?

26. Make up 5 original problems in addition and subtraction of fractions.

MULTIPLICATION OF FRACTIONS

Multiplying a Fraction by a Whole Number

69. How much is 3 times 1 dollar? 4 times 2 dollars?

How much is 3 times 1 eighth? 4 times 2 eighths? 5 times 3 eighths? 6 times 5 eighths?

How much is $3 \times \frac{1}{8}$? $4 \times \frac{2}{8}$? $5 \times \frac{3}{8}$? $6 \times \frac{5}{8}$?

RULE. **To multiply a fraction by a whole number, multiply the numerator by the whole number.**

Give products thus, 3 times $\frac{1}{7}$ equals $\frac{3}{7}$.

1. $3 \times \frac{1}{7}$ $4 \times \frac{1}{5}$ $5 \times \frac{1}{6}$ $6 \times \frac{1}{3}$ $8 \times \frac{1}{4}$

2. $10 \times \frac{1}{6}$ $12 \times \frac{1}{9}$ $8 \times \frac{1}{6}$ $15 \times \frac{1}{3}$ $20 \times \frac{1}{5}$

3. $4 \times \frac{2}{3}$ $5 \times \frac{3}{7}$ $8 \times \frac{3}{4}$ $10 \times \frac{2}{9}$ $12 \times \frac{5}{8}$

Give the products of the following, thus, $\frac{3}{8}$ multiplied by 5 equals $\frac{15}{8}$ or $1\frac{7}{8}$.

4. $\frac{3}{8} \times 5$ $\frac{4}{5} \times 9$ $\frac{7}{9} \times 6$ $\frac{3}{4} \times 7$

5. $\frac{9}{10} \times 9$ $\frac{5}{12} \times 10$ $\frac{9}{11} \times 8$ $\frac{4}{5} \times 9$

6. $\frac{5}{6} \times 4$ $\frac{5}{6} \times 8$ $\frac{5}{6} \times 6$ $\frac{7}{8} \times 3$

WRITTEN EXERCISES

70. 1. Multiply $\$\frac{4}{5}$ by 27.

Process. $27 \times \$\frac{4}{5} = \frac{108}{5} = \$21\frac{3}{5}$ *Ans.*

Multiply:

2. $\$\frac{3}{8}$ by 45.

3. $\frac{5}{9}$ sq. yd. by 200.

4. $\frac{6}{7}$ by 150.

5. $\frac{7}{10}$ by 43.

6. $\frac{5}{16}$ lb. by 57.

7. $1\frac{1}{12}$ ft. by 341.

Sometimes cancelation may serve to shorten the process. (See Art. 36.)

8. Multiply $\frac{5}{16}$ by 100.

First Process. $100 \times \frac{5}{16} = \dfrac{\overset{25}{\cancel{100}} \times 5}{\underset{4}{\cancel{16}}} = \frac{125}{4} = 31\frac{1}{4}.$

Second Process. $\overset{25}{\cancel{100}} \times \dfrac{5}{\underset{4}{\cancel{16}}} = \frac{125}{4} = 31\frac{1}{4}.$

Multiply, using cancelation whenever possible:

9. $\$\frac{24}{25}$ by 60.

10. $1\frac{1}{14}$ by 63.

11. $\frac{21}{40}$ mi. by 160.

12. $\frac{7}{18}$ T. by 30.

13. $\frac{15}{24}$ cu. ft. by 16.

14. $\frac{19}{30}$ gal. by 42.

15. $\frac{17}{20}$ mi. by 12.

16. $\frac{16}{35}$ by 14.

17. $\frac{15}{21}$ bu. by 9.

18. $\frac{35}{36}$ A. by 120.

19. $\$\frac{49}{50}$ by 250.

20. $\frac{5}{24}$ pwt. by 20.

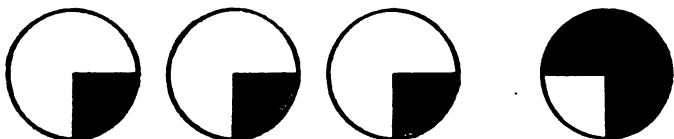### Multiplying a Whole Number by a Fraction

CLASS EXERCISE

71. Each pupil should be provided with circles or rectangles of paper and either scissors or pencils, and should do the exercises called for.

How much is $\frac{1}{2}$ of 4? Of 2? Of 1?

How much is $\frac{1}{4}$ of 8? Of 4?

How much is $\frac{1}{4}$ of 3?

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{1}{4}$ of 3 = 3 times $\frac{1}{4} = \frac{3}{4}$.

Show by means of the pictures or by means of paper disks that

1. $\frac{1}{4}$ of 3 = $\frac{3}{4}$ of 1.
2. $3 \times \frac{1}{4} = \frac{3}{4}$.
3. $\frac{1}{4}$ of 3 = 3 times $\frac{1}{4}$.
4. $\frac{3}{4} = \frac{1}{4}$ of 3.
5. $\frac{3}{4} = 3 \times \frac{1}{4}$.
6. $3 \times \frac{1}{4} = \frac{1}{4}$ of 3.
7. $1 \div 4 = \frac{1}{4}$.
8. $2 \div 4 = \frac{2}{4}$.
9. $3 \div 4 = \frac{3}{4}$.
10. $\frac{3}{4} = 3 \div 4$.

By means of similar pictures or paper disks prove that the following relations are true:

11. 3 times $\frac{3}{4} = \frac{9}{4}$. 12. $\frac{3}{4}$ of 3 = $\frac{9}{4}$. 13. $\frac{3}{4}$ of 3 = $3 \times \frac{3}{4}$.

14. By means of 9 similar pictures or disks, prove that $\frac{3}{4}$ of 3 = $\frac{1}{4}$ of 3 times 3.

These proofs give the following principle:

PRINCIPLE. **To find a fractional part of a number, multiply the number by the fraction.**

That is, $\frac{3}{4}$ of $3 = 3 \times \frac{3}{4}$, or $\frac{3}{4} \times 3$.

Hence in any expression like $\frac{3}{5}$ of 20, the word "of" may be replaced by the multiplication sign without changing the result.

Finding the fractional part of a number is called **multiplying by a fraction.**

For example, when we state $\frac{3}{4}$ of $5 = \frac{15}{4}$, we multiply 5 by the fraction $\frac{3}{4}$.

RULE. **To multiply a whole number by a fraction, multiply the whole number by the numerator for a new numerator and rewrite the old denominator.**

Fractional Parts of Numbers

SIGHT EXERCISES

72. 1. How much is $\frac{1}{2}$ of \$20? of \$18? of \$36? of \$48? of \$70? of \$100? of \$500? of \$1000?

$$\tfrac{1}{2} \text{ of } 20 = 20 \div 2 = 10.$$

2. Find $\frac{1}{3}$ of 24, 30, 48, 72, 69, 75, 150, 225.

3. How much is $\frac{1}{5}$ of 60? 50? 125? 100? 500?

4. How many in $\frac{1}{8}$ of 48? 64? 72? 128? 160?

5. Find $\frac{1}{12}$ of 48, 60, 96, 144, 240, 132, 600, 720, 360, 2400.

73. Explain answers:

1. Think $\frac{1}{4}$ of 20 ct.; how much is $\frac{3}{4}$ of 20 ct.?

Explanation. $\frac{1}{4}$ of 20 = 5; $\frac{3}{4}$ of 20 = 3 times 5 or 15.

2. Think $\frac{1}{4}$ of 24; how much is $\frac{3}{4}$ of 24?

3. Think $\frac{1}{8}$ of 32; find $\frac{3}{8}$ of 32; $\frac{5}{8}$ of 32; $\frac{7}{8}$ of 32.

4. Think $\frac{1}{6}$ of 48; find $\frac{5}{6}$ of 48.

5. Think $\frac{1}{5}$ of 30; find $\frac{2}{5}$ of 30; $\frac{3}{5}$ of 30.

74. How much is $\frac{3}{4}$ of $9?

Use whichever form of explanation you prefer:

Explanation 1	**Explanation 2**
$\frac{1}{4}$ of \$9 = \$$\frac{9}{4}$,	$\frac{3}{4}$ of \$9 = 9 times \$$\frac{1}{4}$
$\frac{3}{4}$ of \$9 = \$$\frac{27}{4}$ or \$6$\frac{3}{4}$.	= \$$\frac{27}{4}$ = \$6$\frac{3}{4}$.

Give answers rapidly, as follows: $\frac{3}{4}$ of \$10 = \$$\frac{30}{4}$ = \$7$\frac{2}{4}$ or \$7$\frac{1}{2}$.

How much is

1. $\frac{3}{4}$ of \$10? $\frac{3}{4}$ of \$11? $\frac{3}{4}$ of \$7? $\frac{3}{4}$ of \$5? $\frac{3}{4}$ of \$15?

2. $\frac{4}{5}$ of 8 ft.? $\frac{4}{5}$ of 9 lb.? $\frac{4}{5}$ of 11 T.? $\frac{4}{5}$ of 12 ' u.? $\frac{4}{5}$ of 6 qt.?

3. $\frac{5}{8}$ of 7? $\frac{4}{9}$ of 11? $\frac{5}{7}$ of 10? $\frac{7}{8}$ of 12? $\frac{9}{10}$ of 6?

75. 1. Find $\frac{11}{24}$ of 84.

First Process. $\frac{11}{24}$ of $84 = \frac{11}{\cancel{24}} \times \cancel{84}^{\,7} = \frac{77}{2} = 38\frac{1}{2}$ *Ans.*
$\qquad\qquad\qquad\;\;_2$

Or we may cancel directly:

Second Process. $\frac{11}{\cancel{24}} \text{ of } \cancel{84}^{\,7} = \frac{77}{2} = 38\frac{1}{2}$ *Ans.*
$\qquad\qquad\;\;_2$

Find results, using cancelation whenever possible:

2. $\frac{11}{16}$ of 160.	**3.** $\frac{11}{25}$ of 50.
4. $\frac{13}{20}$ of 40.	**5.** $\frac{11}{12}$ of 420.
6. $\frac{7}{18}$ of 300.	**7.** $\frac{20}{21}$ of 350.
8. $\frac{5}{16}$ of 12.	**9.** $\frac{9}{20}$ of 30.
10. $\frac{9}{10}$ of 145.	**11.** $\frac{12}{25}$ of 500 ft.
12. $\frac{9}{40}$ of $250.	**13.** $\frac{35}{64}$ of 1728 cu. in.
14. $\frac{374}{500}$ of 7000 gr.	**15.** $\frac{5}{18}$ of 144 sq. in.
16. $\frac{17}{21}$ of 231 cu. in.	**17.** $\frac{73}{120}$ of 5760 gr.
18. $\frac{19}{24}$ of 196 lb.	**19.** $\frac{7}{9}$ of 5280 ft.

Product of a Mixed Number and a Whole Number

76. Multiply $27\frac{2}{3}$ by 31.

Since $27\frac{2}{3} \times 31 = 31 \times 27\frac{2}{3}$ (see Note, p. 14), we may take either number for the multiplicand. Hence, there are two ways to find the product.

·Learn both ways.

A mixed number multiplied by a whole number.	A whole number multiplied by a mixed number.

$$\begin{array}{r} 27\tfrac{2}{3} \\ 31 \end{array}$$

$31 \times \tfrac{2}{3} = 62 \times \tfrac{1}{3} = 3\overline{)62}$

$$20\tfrac{2}{3}$$

$$\begin{array}{rr} 1 \times 27 = & 27 \\ 30 \times 27 = 810 & 81 \\ \hline 31 \times 27\tfrac{2}{3} = & 857\tfrac{2}{3} \; Ans. \end{array}$$

$$\begin{array}{r} 31 \\ 27\tfrac{2}{3} \end{array}$$

$\tfrac{2}{3}$ of $31 = \tfrac{1}{3}$ of $62 = 3\overline{)62}$

$$20\tfrac{2}{3}$$

$$\begin{array}{rr} 7 \times 31 = 217 & 217 \\ 20 \times 31 = 620 & 62 \\ \hline 27\tfrac{2}{3} \times 31 = & 857\tfrac{2}{3} \; Ans. \end{array}$$

WRITTEN EXERCISES

HINT. Time may sometimes be saved by making the larger number the multiplicand.

Find products:

1. $12\tfrac{2}{3} \times 20$ $16\tfrac{1}{2} \times 30$ $18\tfrac{2}{4} \times 25$ $30\tfrac{1}{4} \times 51$

2. $14\tfrac{2}{5} \times 16$ $23\tfrac{3}{8} \times 29$ $64\tfrac{5}{8} \times 80$ $37\tfrac{1}{2} \times 49$

3. $24\tfrac{5}{8} \times 1728$ $34\tfrac{3}{4} \times 196$ $18\tfrac{3}{8} \times 144$ $16\tfrac{2}{3} \times 231$

ORAL EXERCISES

77. If a mixed number contains small figures only, time may sometimes be saved by reducing it to an improper fraction before multiplying; *e.g.* $2\tfrac{2}{3} \times 4 = \tfrac{8}{3} \times 4 = \tfrac{32}{3} = 10\tfrac{2}{3}$.

1. Multiply by 5: $1\tfrac{2}{3}$, $3\tfrac{1}{2}$, $2\tfrac{3}{4}$, $1\tfrac{5}{8}$, $2\tfrac{1}{6}$, $6\tfrac{1}{4}$, $3\tfrac{1}{8}$.

2. Multiply the same mixed numbers by 3; by 4; by 6; by 10.

Multiply, using cancelation wherever possible:

3. $8\frac{2}{3}$ by 6; $9\frac{1}{2}$ by 4; $2\frac{1}{6}$ by 12; $2\frac{4}{5}$ by 9.

4. $2\frac{3}{4}$ by 8; $1\frac{2}{3}$ by 5; $3\frac{5}{6}$ by 4; $3\frac{2}{3}$ by 9.

5. $7\frac{1}{2} \times 3 = ?$ $5\frac{1}{3} \times 4 = ?$ $6\frac{1}{4} \times 8 = ?$

6. $8\frac{1}{5}$ by 7 = ? $3\frac{1}{9}$ by 18 = ? $10\frac{1}{4}$ by 12 = ?

Multiply:

7. 4 by $3\frac{1}{2}$; 2 by $1\frac{1}{4}$; 3 by $2\frac{1}{2}$; 10 by $2\frac{2}{5}$.

8. 6 by $4\frac{1}{3}$; 3 by $2\frac{2}{3}$; 8 by $1\frac{1}{2}$; 12 by $3\frac{1}{4}$.

9. 10 by $2\frac{1}{4}$; 4 by $3\frac{7}{8}$; 5 by $2\frac{1}{5}$; 15 by $2\frac{1}{3}$.

10. 9 by $3\frac{2}{3}$; 12 by $5\frac{1}{6}$; 8 by $3\frac{3}{4}$; 20 by $3\frac{3}{4}$.

WRITTEN EXERCISES

78. Find

1. $\frac{3}{4}$ of 76; $\frac{2}{5}$ of 95; $\frac{3}{8}$ of 104.

2. $\frac{2}{9}$ of 99 da.; $\frac{4}{7}$ of 91 ft.; $\frac{4}{9}$ of 153 lb.

3. $\frac{5}{6}$ of \$252; $\frac{7}{8}$ of \$384; $\frac{5}{12}$ of \$900.

4. $\frac{11}{12}$ of \$4.76; $\frac{17}{30}$ of \$5.00; $\frac{14}{15}$ of \$3.75.

5. $\frac{3}{4}$ of 395; $\frac{2}{5}$ of 621; $\frac{3}{8}$ of 759.

6. $\frac{2}{3}$ of 827; $\frac{7}{8}$ of 961; $\frac{14}{15}$ of 526.

7. $\frac{4}{7}$ of 830 yd.; $\frac{5}{6}$ of 716 ft.; $\frac{11}{12}$ of 493 lb.

8. $\frac{17}{30}$ of \$6.83; $\frac{5}{12}$ of \$13.61; $\frac{4}{9}$ of \$24.50.

Multiply:

9. $10\frac{1}{3}$ by 12; $15\frac{1}{2}$ by 14; $21\frac{1}{3}$ by 18.

10. $7\frac{5}{7}$ by 21; $9\frac{7}{9}$ by 36; $12\frac{5}{8}$ by 15.

11. $35 by $7\frac{2}{5}$; $29 by $5\frac{3}{4}$; $20 by $6\frac{3}{8}$.

12. 40 by $3\frac{4}{5}$; 23 by $9\frac{1}{2}$; 25 by $8\frac{1}{4}$.

13. $18\frac{1}{2}$ by 6; $24\frac{2}{3}$ by 10; $30\frac{1}{2}$ by 20.

Find products:

14. $15 \times 13\frac{1}{2} = ?$ $100 \times 16\frac{1}{2} = ?$ 250 by $12\frac{1}{2} = ?$

15. $24 \times 17\frac{2}{3} = ?$ $19 \times 19\frac{5}{8} = ?$ $31 \times 30\frac{1}{5} = ?$

16. $61 \times 25\frac{1}{4} = ?$ $52 \times 79\frac{1}{8} = ?$ $38 \times 104\frac{4}{7} = ?$

17. $3\frac{29}{60} \times 182 = ?$ $47 \times 35\frac{4}{11} = ?$ $68\frac{1}{13} \times 23 = ?$

18. $42\frac{7}{25} \times 67 = ?$ $30 \times 40\frac{3}{10} = ?$ $100 \times 50\frac{7}{8} = ?$

19. $76\frac{1}{4} \times 326$ bbl. $= ?$ $29\frac{3}{5} \times 465$ yd. $= ?$

20. $37\frac{3}{4} \times $400 = ?$ $26\frac{1}{3} \times $57.90 = ?$

79. 1. How many are $\frac{3}{5}$ of 120 da. ?

2. 360 apples are divided into 2 equal piles. How many apples are in each pile ?

3. A farmer packs 90 doz. eggs into 3 baskets, putting an equal number into each basket. How many eggs in each basket?

4. The distance between New York and Albany is 143 mi. The distance between New York and Poughkeepsie is $\frac{1}{2}$ as great. What is the distance between New York and Poughkeepsie?

5. An express wagon carried packages that weighed 250 lb. After some of the packages had been delivered, the load was only $\frac{3}{5}$ as heavy as at first. How heavy were the packages that were delivered? How heavy were the packages that were still in the wagon?

6. There were 400 bbl. of sugar on a boat. $\frac{1}{8}$ of them contained white sugar; the remainder contained brown sugar. Find the number of barrels of each kind.

7. A girl receives $2 a day. How much will she earn in $8\frac{1}{4}$ da.?

8. A machine polishes 6 pencils a minute. How many will it polish in 15 min.? In $18\frac{1}{2}$ min.?

9. If a train travels 20 mi. an hour, how far will it go in $3\frac{1}{2}$ hr. at that rate? In $4\frac{1}{4}$ hr.?

10. A workman makes an average of $4\frac{1}{2}$ whistles an hour. How many whistles does he make in 6 hr.? In 8 hr.?

11. The furnaces in a factory burn an average of $10\frac{1}{2}$ T. of coal a day. How many tons will they burn, at that rate, in 4 da.? In 8 da.?

12. In a factory, the number of men employed is $3\frac{1}{2}$ times as great as the number of boys. If there are 26 boys, how many men are employed?

WRITTEN PROBLEMS

80. 1. The distance from A to C is 153 ft. The distance from A to B is $\frac{1}{3}$ of the distance from A to C.

(*a*) Find the distance from A to B.

(*b*) Find the distance from B to C.

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A              B                             C
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2. One baseball team wins 68 games. Another team wins $\frac{1}{4}$ as many. How many games are won by the second team?

3. There are 48 pupils in one class and $\frac{3}{4}$ as many in another class. How many pupils in the second class?

4. 120 children went for a sail. $\frac{7}{8}$ of them were girls.

(*a*) How many girls? (*b*) How many boys?

5. George weighs $\frac{8}{9}$ as much as Will. Will weighs 111 lb. How much does George weigh?

6. Find the weight of twelve desks if each desk weighs $18\frac{3}{4}$ lb.

7. How much money can a man earn in 52 wk., if he is paid $\$23\frac{1}{4}$ a week?

8. A girl is paid $\$\frac{4}{5}$ a day. How much does she earn in 36 da.?

9. If a train travels $30\frac{5}{6}$ mi. an hour, how far will it travel in 9 hr.?

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10. How many pounds are there in $\frac{4}{5}$ of a ton of coal? (2000 lb. = 1 T.)

11. How many pounds in $\frac{7}{8}$ of a barrel of flour?
(196 lb. = 1 bbl.)

12. How many pencils in $27\frac{5}{6}$ doz.?

13. A street is 250 ft. long. The width is $\frac{3}{10}$ of the length. Find the width.

14. A baker uses $24\frac{7}{8}$ lb. of flour each day. How many pounds does he use in 42 days at that rate?

15. An agent sells:

> 362 tickets at $1\frac{1}{2}$ each.
> 276 tickets at $1\frac{1}{4}$ each.
> 412 tickets at $\frac{3}{4}$ each.

(*a*) How much money is received for each kind of ticket?

(*b*) How much money is received in all?

Find the cost of the following:

16. 340 books at $1\frac{3}{4}$ each.

17. 270 pictures at $1\frac{1}{2}$ each.

18. 84 lamps at $\frac{4}{5}$ each.

19. 14 rugs at $25\frac{3}{5}$ each.

20. 3 doz. dresses at $2\frac{3}{4}$ each.

21. Make up five original problems in the multiplication of mixed numbers by whole numbers.

Product of a Fraction by a Fraction

For Reading and Discussion

81. How many 12ths in $\frac{1}{2}$? In $\frac{1}{3}$? In $\frac{1}{4}$? In $\frac{1}{6}$? In $\frac{2}{3}$? In $\frac{3}{4}$? In $\frac{5}{6}$?

Looking at the picture, answer the following questions:

What part of the whole rectangle is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{2}$ of $\frac{2}{3}$? $\frac{1}{4}$ of $\frac{2}{3}$? $\frac{1}{3}$ of $\frac{3}{4}$? $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{1}{2}$ of $\frac{5}{6}$?

Putting these results in a table we have:

$$\frac{1}{2} \text{ of } \frac{1}{2} = \frac{1}{4} \qquad\qquad \frac{1}{2} \text{ of } \frac{2}{3} = \frac{2}{6} \text{ or } \frac{1}{3}$$

$$\frac{1}{2} \text{ of } \frac{1}{3} = \frac{1}{6} \qquad\qquad \frac{1}{4} \text{ of } \frac{2}{3} = \frac{2}{12} \text{ or } \frac{1}{6}$$

$$\frac{1}{3} \text{ of } \frac{1}{4} = \frac{1}{12} \qquad\qquad \frac{1}{3} \text{ of } \frac{3}{4} = \frac{3}{12} \text{ or } \frac{1}{4}$$

$$\frac{1}{2} \text{ of } \frac{1}{6} = \frac{1}{12} \qquad\qquad \frac{2}{3} \text{ of } \frac{3}{4} = \frac{6}{12} \text{ or } \frac{1}{2}$$

$$\frac{1}{2} \text{ of } \frac{5}{6} = \frac{5}{12}$$

We see that in each example the answer may be found by multiplying together the numerators of the fractions for a new numerator, and the denominators for a new denominator.

In Art. 71, it was shown that $\frac{1}{8}$ of $3 = \frac{1}{8} \times 3$. Likewise $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{2} \times \frac{1}{2}$; $\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{3} \times \frac{3}{4}$, etc.

RULE. **To multiply a fraction by a fraction, multiply the numerators together for a new numerator and the denominators for a new denominator.**

Instead of actually multiplying, it is better to indicate the operation and cancel if possible.

Multiply $\frac{8}{9}$ by $\frac{3}{10}$.

Process. $\overset{4}{\underset{3}{\cancel{\frac{8}{9}}}} \times \overset{}{\underset{5}{\frac{\cancel{3}}{\cancel{10}}}} = \frac{4}{15}.$

ORAL EXERCISES

82. 1. Multiply $\frac{1}{2}$ by $\frac{2}{3}$; $\frac{1}{4}$ by $\frac{4}{5}$; $\frac{1}{2}$ by $\frac{6}{7}$.

2. Multiply $\frac{3}{5}$ by $\frac{10}{21}$; $\frac{4}{7}$ by $\frac{7}{8}$; $\frac{5}{8}$ by $\frac{12}{13}$.

3. $\frac{5}{6}$ of $\frac{3}{4} = ?$; $\frac{5}{6}$ of $\frac{8}{9} = ?$; $\frac{5}{6}$ of $\frac{11}{12} = ?$

4. $\frac{1}{2}$ of $\frac{3}{4} = ?$; $\frac{3}{8}$ of $\frac{9}{10} = ?$; $\frac{2}{5}$ of $\frac{4}{7} = ?$

Find the products:

5. $\frac{7}{12} \times \frac{6}{7}$; $\frac{3}{5} \times \frac{9}{10}$; $\frac{3}{8} \times \frac{7}{9}$; $\frac{8}{9} \times \frac{3}{4}$; $\frac{5}{6} \times \frac{9}{10}$.

6. $\frac{4}{5} \times \frac{3}{8}$; $\frac{3}{8} \times \frac{8}{27}$; $\frac{4}{7} \times \frac{5}{9}$; $\frac{5}{8} \times \frac{6}{7}$; $\frac{4}{5} \times \frac{5}{6}$.

7. $\frac{8}{11} \times \frac{2}{3}$; $\frac{11}{15} \times \frac{25}{22}$; $\frac{14}{15} \times \frac{25}{28}$; $\frac{24}{25} \times \frac{15}{36}$.

WRITTEN EXERCISES

83. Multiply:

1. $\frac{7}{10}$ by $\frac{9}{16}$; $\frac{8}{9}$ by $\frac{11}{12}$; $\frac{7}{10}$ by $\frac{20}{49}$; $\frac{16}{21}$ by $\frac{9}{20}$.

2. $\frac{15}{22}$ by $\frac{11}{60}$; $\frac{25}{32}$ by $\frac{8}{75}$; $\frac{21}{25}$ by $\frac{13}{14}$; $\frac{3}{7}$ by $\frac{35}{36}$.

3. $\frac{16}{21}$ by $\frac{14}{9}$; $\frac{23}{30}$ by $\frac{9}{8}$; $\frac{27}{30}$ by $\frac{15}{2}$; $\frac{15}{16}$ by $\frac{24}{35}$.

4. $\frac{9}{14}$ of $\frac{21}{45}$; $\frac{12}{17}$ of $\frac{51}{108}$; $\frac{15}{16}$ of $\frac{64}{90}$; $\frac{8}{9}$ by $\frac{63}{64}$.

5. $\frac{5}{7}$ of $\frac{56}{85}$; $\frac{11}{18}$ of $\frac{90}{23}$; $\frac{15}{22}$ of $\frac{88}{17}$; $\frac{5}{6}$ by $\frac{24}{25}$.

ORAL PROBLEMS

74. 1. A yard of ribbon costs $\$\frac{3}{4}$. How much does $\frac{1}{2}$ yd. cost? $\frac{1}{4}$ yd.? $\frac{1}{8}$ yd.?

2. An advertising sign is painted on a fence. The capital letters are $\frac{1}{2}$ ft. high; the small letters are $\frac{3}{8}$ as high as the capital letters. How high are the small letters?

3. A plot of ground is $\frac{7}{8}$ mi. long. Its length is divided into 4 equal parts. How long is each part?

4. A piece of cloth $\frac{3}{4}$ of a yard long is cut into 5 pieces of equal length. How long is each piece?

5. A log of wood is $\frac{9}{10}$ ft. thick. It is sawed into six planks of equal thickness. How thick is each plank? What is the total thickness of 3 of the planks?

6. 24 picture post cards weigh $\frac{3}{8}$ lb. How much do 6 cards weigh? 8 cards? 12 cards? 1 card?

7. A boy earns $\$\frac{4}{5}$ a day.
(a) How much does he earn in $\frac{1}{2}$ day?
(b) How much does he earn in $\frac{1}{4}$ day?

8. The distance between 2 farms is $\frac{7}{8}$ of a mile. Find $\frac{2}{3}$ of the distance.

9. A line $\frac{5}{6}$ ft. long is divided into 3 equal parts. How long is each part?

Find the cost of

10. $\frac{1}{2}$ yd. of lace at $\$\frac{3}{4}$ a yard.

11. $\frac{3}{4}$ doz. eggs at $\$\frac{2}{3}$ a dozen.

12. $\frac{1}{4}$ lb. butter at $\$\frac{1}{2}$ a pound.

WRITTEN EXERCISES

85. 1. Multiply $\frac{5}{6}$ by $\frac{3}{10}$; $\frac{3}{4}$ by $\frac{4}{5}$; $\frac{5}{8}$ by $\frac{9}{10}$; $\frac{5}{7}$ by $\frac{14}{20}$.

2. Multiply $1\frac{2}{3}$ by $\frac{1}{4}$.

Explanation Reducing the mixed number to an improper fraction, we have $1\frac{2}{3} = \frac{5}{3}$; $\frac{5}{3} \times \frac{1}{4} = \frac{5}{12}$ *Ans.*

Multiply:

3. $2\frac{1}{2}$ by $\frac{3}{4}$; $3\frac{3}{4}$ by $\frac{5}{6}$; $2\frac{3}{5}$ by $\frac{2}{3}$; $3\frac{1}{3}$ by $\frac{5}{6}$.

4. $4\frac{4}{5}$ by $\frac{2}{7}$; $2\frac{2}{9}$ by $\frac{2}{3}$; $8\frac{1}{3}$ by $\frac{4}{5}$; $1\frac{3}{4}$ by $\frac{5}{8}$.

Sometimes both multiplier and multiplicand are mixed numbers and we have to reduce both of them to improper fractions.

Multiply $2\frac{2}{3}$ by $2\frac{1}{2}$.

$$2\frac{2}{3} \times 2\frac{1}{2} = \frac{8}{3} \times \frac{5}{2} = \frac{40}{6} = 6\frac{4}{6} \text{ or } 6\frac{2}{3}.$$

86. Find products:

1. $3\frac{3}{4} \times 2\frac{3}{5}$.
2. $4\frac{3}{4} \times 2\frac{1}{5}$.
3. $1\frac{3}{4} \times 2\frac{1}{3}$.
4. $3\frac{1}{3} \times 2\frac{1}{2}$.
5. $5\frac{1}{6} \times 1\frac{1}{3}$.
6. $3\frac{2}{3} \times 1\frac{1}{2}$.
7. $2\frac{3}{4}$ by $4\frac{7}{8}$.
8. $5\frac{1}{6}$ by $3\frac{2}{3}$.
9. $2\frac{7}{8}$ by $4\frac{1}{5}$.
10. $5\frac{1}{2}$ by $3\frac{5}{6}$.
11. $7\frac{3}{8}$ by $5\frac{2}{5}$.
12. $6\frac{1}{6}$ by $3\frac{2}{3}$.
13. $9\frac{1}{3}$ by $6\frac{3}{8}$.
14. $4\frac{3}{4}$ by $2\frac{7}{8}$.
15. $5\frac{3}{8}$ by $2\frac{7}{8}$.
16. $6\frac{1}{4}$ by $6\frac{1}{4}$.
17. $10\frac{1}{2}$ by $8\frac{1}{4}$.
18. $12\frac{1}{6}$ by $\frac{4}{7}$.
19. $3\frac{9}{10}$ by $5\frac{1}{4}$.
20. $12\frac{1}{4}$ by $12\frac{1}{2}$.
21. $16\frac{2}{3}$ by $\frac{9}{10}$.

87. 1. Multiply: $\frac{1}{2}$ by $5\frac{1}{2}$; $\frac{1}{3}$ by $2\frac{1}{2}$; $\frac{1}{4}$ by $1\frac{1}{5}$.

2. Solve: $\frac{2}{3} \times 4\frac{1}{2} = ?$ $\frac{1}{6} \times 2\frac{1}{2} = ?$ $\frac{3}{4} \times 1\frac{1}{4} = ?$

3. Solve: $\frac{3}{5}$ of $1\frac{2}{3} = ?$ $\frac{4}{9}$ of $1\frac{1}{3} = ?$ $\frac{2}{5}$ of $2\frac{1}{3} = ?$

4. Multiply: $3\frac{1}{4}$ by $\frac{1}{3}$; $4\frac{1}{2}$ by $\frac{1}{2}$; $1\frac{1}{2}$ by $\frac{2}{3}$.

5. Multiply: $6\frac{1}{2}$ by $\frac{2}{3}$; $4\frac{2}{3}$ by $\frac{3}{4}$; $7\frac{1}{2}$ by $\frac{1}{3}$.

6. Solve: $9\frac{1}{2} \times \frac{1}{2} = ?$ $5\frac{1}{3} \times \frac{1}{6} = ?$ $2\frac{1}{2} \times \frac{3}{5} = ?$

88. Multiply:

1. $1\frac{1}{5}$ by $1\frac{1}{2}$; $1\frac{1}{3}$ by $1\frac{1}{3}$; $1\frac{1}{2}$ by $1\frac{1}{8}$; $1\frac{1}{3}$ by $1\frac{1}{4}$.
2. $2\frac{1}{2}$ by $1\frac{1}{3}$; $2\frac{1}{3}$ by $1\frac{1}{3}$; $2\frac{1}{5}$ by $1\frac{1}{4}$; $2\frac{2}{3}$ by $1\frac{1}{2}$.
3. $1\frac{1}{2} \times 1\frac{1}{4}$; $2\frac{1}{2} \times 2\frac{1}{2}$; $2\frac{1}{4} \times 2\frac{1}{2}$; $3\frac{1}{3}$ by $1\frac{1}{5}$.
4. $3\frac{1}{3} \times 1\frac{1}{2}$; $3\frac{1}{2} \times 3\frac{1}{2}$; $3\frac{1}{4} \times 1\frac{1}{4}$; $2\frac{1}{4}$ by $3\frac{1}{3}$.
5. $4\frac{1}{3} \times 1\frac{1}{4}$; $3\frac{1}{4} \times 1\frac{1}{2}$; $1\frac{3}{8} \times 1\frac{5}{8}$; $1\frac{3}{8}$ by $1\frac{3}{8}$.

89. Multiply :

1. $\frac{5}{18}$ by $7\frac{1}{5}$; $\frac{9}{16}$ by $12\frac{1}{3}$; $\frac{7}{24}$ by $15\frac{1}{2}$.

2. $\frac{21}{25}$ by $18\frac{1}{3}$; $\frac{42}{45}$ by $6\frac{1}{5}$; $\frac{19}{27}$ by $12\frac{1}{5}$

3. $16\frac{1}{3}$ by $\frac{6}{7}$; $25\frac{1}{2}$ by $\frac{9}{17}$; $30\frac{1}{4}$ by $\frac{6}{7}$.

4. $36\frac{1}{2}$ by $\frac{4}{5}$; $45\frac{1}{4}$ by $\frac{6}{7}$; $18\frac{3}{4}$ by $\frac{8}{15}$.

Solve :

5. $\frac{5}{18}$ of $27\frac{2}{3}=$? $\frac{6}{7}$ of $12\frac{4}{9}=$? $\frac{9}{10}$ of $16\frac{2}{3}=$?

6. $15\frac{1}{3} \times \frac{9}{10}=$? $13\frac{1}{6} \times \frac{14}{15}=$? $14\frac{2}{5} \times \frac{8}{9}=$?

90. Multiply :

1. $6\frac{1}{4}$ by $7\frac{1}{3}$; $8\frac{1}{2}$ by $3\frac{4}{7}$; $12\frac{1}{2}$ by $5\frac{5}{8}$.

2. $15\frac{1}{2}$ by $15\frac{1}{2}$; $22\frac{2}{3}$ by $5\frac{1}{2}$; $30\frac{3}{4}$ by $4\frac{1}{4}$.

3. $32\frac{1}{8}$ by $40\frac{1}{4}$; $29\frac{3}{4}$ by $14\frac{7}{8}$; $20\frac{6}{7}$ by $15\frac{1}{8}$.

4. $18\frac{4}{5}$ by $21\frac{9}{10}$; $16\frac{3}{4}$ by $12\frac{5}{8}$; $19\frac{3}{4}$ by $15\frac{6}{7}$.

Solve :

5. $12\frac{1}{2} \times 12\frac{1}{2}=$?   8. $20\frac{7}{8} \times 30\frac{5}{6}=$?

6. $15\frac{3}{4} \times 18\frac{5}{6}=$?   9. $27\frac{3}{5} \times 12\frac{2}{3}=$?

7. $16\frac{3}{8} \times 15\frac{1}{2}=$?   10. $40\frac{5}{6} \times 40\frac{5}{6}=$?

Work mentally, using pencil only when the numbers are too large to be carried in the mind.

91. 1. How much is $\frac{1}{2}$ of $9\frac{1}{2}$?

2. If a ton of coal costs $\$4\frac{3}{4}$, how much will $\frac{1}{2}$ of a ton cost ?

3. A board is $10\frac{1}{2}$ ft. long. $\frac{1}{4}$ of it is used to make a shelf. How long is the shelf? How much wood is not used?

4. If a dozen caps cost $\$5\frac{1}{2}$, how much will half a dozen cost? A quarter of a dozen?

5. A tailor receives $\$\frac{2}{5}$ an hour. How much will he earn in 10 hr.?

6. Vinegar costs $7\frac{1}{2}$ ct. a quart. Find the cost of 30 gal.

7. Find the cost of $6\frac{1}{2}$ ft. of lumber at $5\frac{1}{2}$ ct. a foot.

8. George walked $2\frac{1}{2}$ mi. an hour for $2\frac{1}{2}$ hr. How many miles did he walk?

9. A printing press printed an average of $12\frac{1}{2}$ papers a minute. How many does it print each hour?

10. The profit on a dozen blank books is $2\frac{1}{2}$ ct. How much is the profit on $7\frac{1}{2}$ doz.?

11. Find the cost of 12 boxes of window glass at $\$2\frac{1}{2}$ a box.

12. The duty on cocoa is $2\frac{1}{2}$ ct. a lb. Find the duty on $10\frac{1}{2}$ lb.

13. A school desk weighs $24\frac{3}{4}$ lb. Find the weight of 3 doz. desks.

PROBLEMS

92. 1. The distance from A to C is $154\frac{1}{2}$ ft. The distance from A to B is $\frac{1}{5}$ the distance from A to C.

  (*a*) Find the distance from A to B.

  (*b*) Find the distance from B to C.

2. If the distance from A to C were $190\frac{1}{2}$ ft., and the distance from A to B were $\frac{2}{5}$ of the distance from A to C,

  (*a*) How far would it be from A to B?

  (*b*) How far would it be from B to C?

(Draw a diagram.)

3. \$ $60\frac{3}{5}$ is divided equally among 4 boys. How much should each one receive ?

4. \$75 is to be divided among A, B, and C. A is to receive $\frac{1}{2}$ of the money; B, $\frac{1}{4}$; C, the remainder. How much should each one receive ?

5. 18 packages weigh $170\frac{5}{8}$ lb. If the packages are of equal weight, how much does 1 package weigh ? 3 packages ? 6 packages ?

6. Find the weight of 37 packages of sugar, if each package weighs $3\frac{1}{2}$ lb.

7. 5 boys form a basketball team. Their average weight is $118\frac{3}{4}$ lb. Find the total weight of all.

8. An express train travels at the rate of $40\frac{7}{8}$ mi. an hour.

(a) How far does the train travel in 12 hr. ?

(b) How far does the train travel in $14\frac{3}{4}$ hr. ?

9. An express train travels at the rate of $43\frac{3}{4}$ mi. an hour. A local train travels at the rate of $36\frac{7}{8}$ mi. an hour.

(a) How many more miles does the express train travel in 9 hours than the local train?

(b) How many more miles does the express train travel in $11\frac{3}{4}$ hours ?

10. Find the weight of $89\frac{3}{4}$ bu. of wheat. (Each bushel weighs 60 lb.)

11. 75 bicycles weigh $12\frac{3}{4}$ lb. each. Each bicycle is packed in a wooden crate. Each crate weighs $2\frac{1}{8}$ lb. Find the total weight of the bicycles and the crate.

12. 3,672 oranges were shipped from Florida to New York. The average weight of the oranges was $\frac{5}{8}$ lb. The oranges were packed in boxes; each box weighed $1\frac{1}{8}$ lb. There were 24 oranges in each box. Find the total weight of the oranges and the boxes.

13. The oranges were sold at the rate of $1.08\frac{1}{2}$ a box. Find the value of the oranges.

14. A freight car carried the following:
84 bbl. of flour, weighing 196 lb. each.
72 crates of berries, weighing $36\frac{7}{8}$ lb. each.

60 baskets of peaches, weighing $50\frac{1}{4}$ lb. each.

144 boxes of oranges, weighing $23\frac{5}{8}$ lb. each.

(*a*) Find the total weight.

(*b*) How much less did the oranges weigh than the flour?

15. A manufacturer paid the following:

26 men for working $7\frac{1}{2}$ hr. each at $\$\frac{2}{5}$ per hour.

35 women for working $6\frac{5}{6}$ hr. each at $\$\frac{3}{10}$ per hour.

40 boys for working $9\frac{1}{4}$ hr. each at $\$\frac{3}{20}$ per hour.

50 girls for working $8\frac{3}{4}$ hr. each at $\$\frac{1}{10}$ per hour.

(*a*) How much money was paid to each group?

(*b*) How much money was paid to all?

16. A boat on a lake makes a trip from A to B to C to D to E and a trip back, each day.

The distance from A to B is $3\frac{1}{8}$ mi., from B to C $4\frac{1}{2}$ mi., from C to D $5\frac{3}{8}$ mi , from D to E $4\frac{3}{4}$ mi.

How many miles does the steamboat travel in 90 complete trips (90 da.) ?

Find the cost of the following:

17. $3\frac{3}{4}$ yd. of cloth at $\$1\frac{1}{5}$ a yard.

18. $8\frac{1}{2}$ yd. of silk at $\$3\frac{4}{5}$ a yard.

19. $15\frac{3}{4}$ yd. of lace at $\$2\frac{1}{2}$ a yard.

20. $12\frac{1}{2}$ yd. of linen at $\$1\frac{3}{4}$ a yard.

21. $34\frac{1}{2}$ yd. of carpet at $\$2\frac{1}{4}$ a yard.

ORAL EXERCISES

93. Find products:

1. $9 \times \frac{1}{3}$
2. $8 \times \frac{1}{4}$
3. $6 \times \frac{1}{2}$
4. $12 \times \frac{2}{3}$
5. $16 \times \frac{3}{4}$
6. $21 \times \frac{3}{7}$
7. $45 \times \frac{1}{9}$
8. $28 \times \frac{5}{7}$
9. $36 \times \frac{5}{6}$
10. $18 \times \frac{2}{3}$

11. $\frac{3}{5} \times 25$
12. $\frac{2}{9} \times 18$
13. $\frac{1}{7} \times 28$
14. $\frac{3}{4} \times 24$
15. $\frac{2}{3} \times 33$
16. $\frac{5}{6} \times 48$
17. $\frac{3}{8} \times 24$
18. $\frac{4}{5} \times 50$
19. $\frac{1}{12} \times 84$
20. $\frac{5}{12} \times 144$

21. $\frac{2}{9}$ of 45
22. $\frac{5}{6}$ of 42
23. $\frac{3}{8}$ of 64
24. $\frac{4}{7}$ of 63
25. $\frac{3}{5}$ of 40
26. $\frac{1}{12}$ of 108
27. $\frac{7}{9}$ of 108
28. $\frac{4}{5}$ of 65
29. $\frac{9}{10}$ of 70
30. $\frac{8}{9}$ of 81

ORAL EXERCISES

Find products:

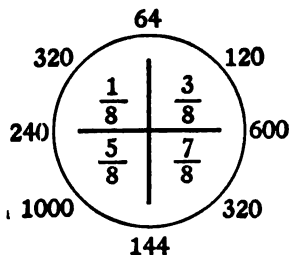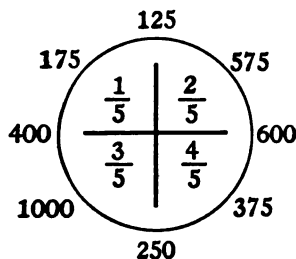|		
1. $\frac{2}{3} \times \frac{3}{4}$	11. $\frac{5}{8}$ of 25	21. $\frac{2}{3} \times 1\frac{1}{2}$
2. $\frac{3}{5} \times \frac{5}{6}$	12. $\frac{4}{7}$ of 14	22. $\frac{1}{6} \times 2\frac{2}{5}$
3. $\frac{1}{2} \times \frac{7}{8}$	13. $\frac{7}{12}$ of 11	23. $\frac{2}{9} \times 1\frac{2}{7}$
4. $\frac{1}{4} \times \frac{5}{9}$	14. $\frac{9}{10}$ of 13	24. $\frac{1}{8} \times 2\frac{2}{5}$
5. $\frac{1}{8} \times \frac{1}{3}$	15. $\frac{2}{3}$ of 10	25. $\frac{1}{4} \times 1\frac{1}{6}$
6. $\frac{1}{5} \times \frac{1}{6} \times \frac{1}{4}$	16. $\frac{5}{8}$ of 15	26. $\frac{3}{5}$ of $1\frac{2}{3}$
7. $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{3}$	17. $\frac{1}{2}$ of 27	27. $\frac{2}{7}$ of $1\frac{3}{4}$
8. $\frac{1}{5} \times \frac{1}{2} \times \frac{1}{3}$	18. $\frac{3}{10}$ of 15	28. $\frac{2}{5}$ of $2\frac{1}{2}$
9. $\frac{3}{4} \times \frac{2}{5} \times \frac{1}{4}$	19. $\frac{2}{7}$ of 18	29. $\frac{3}{4}$ of $2\frac{1}{3}$
10. $\frac{1}{8} \times \frac{3}{5} \times \frac{5}{6}$	20. $\frac{2}{5}$ of 21	30. $\frac{5}{8}$ of $3\frac{1}{2}$

ORAL EXERCISES

94. 1. $2 \times 1\frac{1}{2} = ?$		11. $1\frac{1}{2} \times 1\frac{1}{2} = ?$
2. $3 \times 1\frac{1}{4} = ?$		12. $1\frac{1}{2} \times 1\frac{2}{3} = ?$
3. $2 \times 2\frac{1}{2} = ?$		13. $1\frac{1}{2} \times 2\frac{1}{2} = ?$
4. $4 \times 1\frac{4}{5} = ?$		14. $2\frac{1}{2} \times 2\frac{1}{2} = ?$
5. $6 \times 2\frac{1}{3} = ?$		15. $1\frac{1}{3} \times 2\frac{1}{3} = ?$
6. $1\frac{1}{2} \times 4 = ?$		16. $1\frac{1}{4} \times 1\frac{1}{4} = ?$
7. $2\frac{1}{2} \times 6 = ?$		17. $1\frac{1}{4} \times 2\frac{1}{2} = ?$
8. $1\frac{1}{3} \times 6 = ?$		18. $1\frac{4}{5} \times 1\frac{1}{4} = ?$
9. $2\frac{1}{3} \times 6 = ?$		19. $2\frac{1}{3} \times 1\frac{2}{3} = ?$
10. $3\frac{1}{2} \times 4 = ?$		20. $4\frac{1}{2} \times 2\frac{1}{2} = ?$

21. $\frac{1}{3} \times 3 \times \frac{1}{2} = ?$

22. $\frac{1}{4} \times \frac{1}{2} \times 6 = ?$

23. $\frac{2}{3} \times \frac{1}{2} \times 9 = ?$

24. $\frac{1}{4} \times 8 \times \frac{2}{3} = ?$

25. $12 \times \frac{1}{2} \times \frac{2}{3} = ?$

26. $15 \times \frac{1}{4} \times \frac{1}{5} = ?$

27. $\frac{3}{8} \times \frac{4}{9} \times 10 = ?$

28. $\frac{2}{7} \times \frac{3}{8} \times 8 = ?$

29. $\frac{5}{9} \times \frac{4}{5} \times 9 = ?$

30. $\frac{1}{2} \times \frac{1}{6} \times \frac{1}{4} \times 12 = ?$

SIGHT DRILL

95. Multiply the numbers on the circumference by each fraction in turn.

NOTE TO TEACHER. Various forms of "Fixing Devices," such as those in Book I, pages 73, 74, 75, 76, 124, 125, 126, etc., are as valuable for daily drills in fractions, as in fundamental operations with integers.

32
80
40
56
64
88
8
48
24
16
72
144
x $\frac{3}{8}$

SIGHT DRILL

"The Ladder"

96. 1. Begin at the lowest step; go up as quickly as possible.
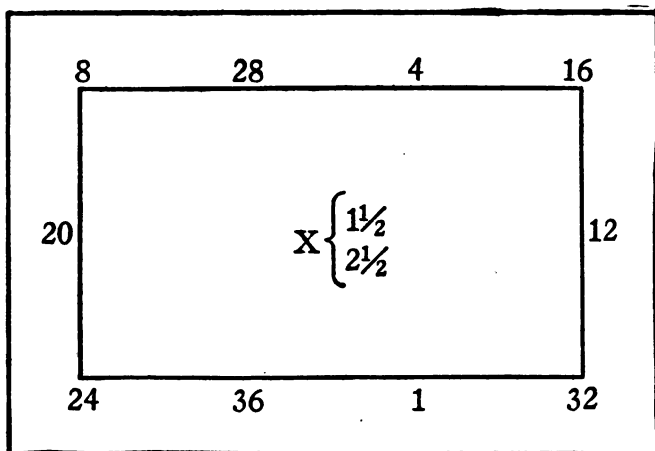
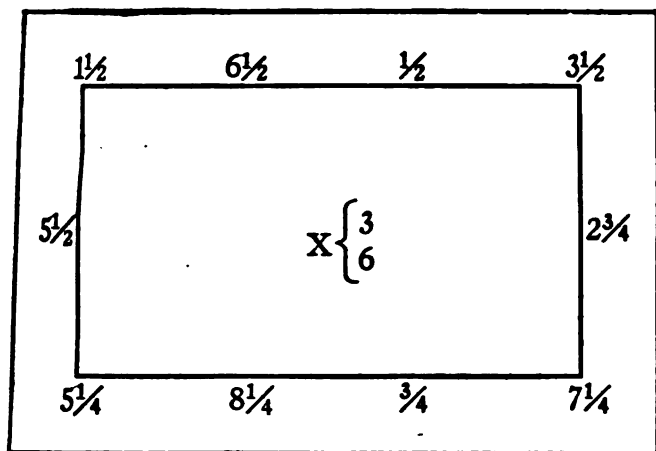**2.** Begin at the highest step; go down as quickly as possible.

3. Begin at the middle and skip about.

4. Write $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$ in place of $\frac{3}{8}$. Give the products quickly.

5. Make other ladders. Use your own numbers. Multiply.

SIGHT DRILL

8	28	4	16
20	$X \begin{cases} 1\frac{1}{2} \\ 2\frac{1}{2} \end{cases}$		12
24	36	1	32

97. Multiply each number in the frame by each number in the rectangle. State the products quickly.
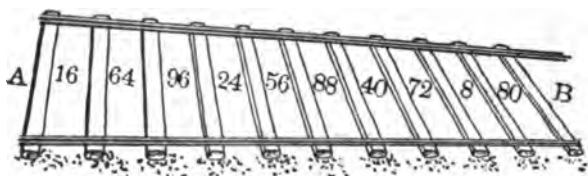
SIGHT DRILL

"The Stairs"

98. State the products quickly.

H

"Railroad Ties"

99. Go from A to B as quickly as possible; then from B to A.

Multiply by $1\frac{1}{2}$, $2\frac{1}{4}$, $2\frac{1}{2}$, $3\frac{1}{4}$.

COMPOUND FRACTIONS

For Reading

100. A compound fraction is a fraction of a fraction; *e.g.* $\frac{2}{3}$ of $\frac{5}{8}$, $\frac{4}{5}$ of $6\frac{2}{3}$.

Since the word "of" in a compound fraction may be replaced by the sign of multiplication, fractions having the multiplication sign between them are sometimes called compound fractions; *e.g.* $\frac{2}{3} \times \frac{5}{8}$, $\frac{4}{5} \times 6\frac{2}{3}$.

Compound fractions are reduced to simple fractions by performing the operations indicated.

WRITTEN EXERCISES

1. Reduce $\frac{3}{8}$ of $\frac{5}{6}$ to simple form.

Process. $\dfrac{\cancel{3}}{8}$ of $\dfrac{5}{\underset{2}{\cancel{6}}} = \dfrac{5}{16}$ *Ans.*

Simplify:

2. $\frac{3}{4}$ of $\frac{7}{15}$.

3. $\frac{8}{9}$ of $\frac{2}{3}$ of 27.

4. $\frac{1}{2}$ of $\frac{5}{6}$ of $12\frac{1}{2}$.

5. $\frac{3}{4}$ of $\frac{2}{3}$ of $700.
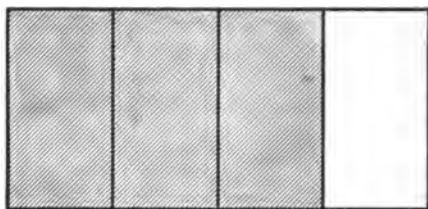
6. $\frac{3}{200} \times 3 \times 1600.

7. $\frac{5}{7}$ of $\frac{3}{11}$ of 231 in.

DIVISION OF FRACTIONS

Dividing a Fraction by a Whole Number

1st Case: Dividing the Numerator

101. If we divide 3 fourths of an apple equally among 3 boys, what part of the apple will each boy receive?

How much is $\frac{3}{4} \div 3$?

$$\frac{3}{4} \div 3 = \frac{3 \div 3}{4} = \frac{1}{4}.$$

How much is $\frac{5}{9} \div 5$? $\frac{8}{9} \div 2$? $\frac{6}{7} \div 3$?

$$3/4 \div 3 = 1/4$$

PRINCIPLE. **A fraction may be divided by a whole number by dividing the numerator by the whole number, the denominator remaining the same.**

ORAL EXERCISES

102. Give the quotients rapidly:

1. $\frac{9}{10} \div 3$
2. $\frac{12}{15} \div 4$
3. $\frac{6}{7} \div 2$
4. $\frac{16}{19} \div 8$

5. $\frac{20}{21} \div 5$
6. $\frac{24}{25} \div 6$
7. $\frac{15}{16} \div 3$
8. $\frac{35}{36} \div 7$

This method of dividing a fraction by a whole number is used whenever the whole number is an exact divisor of the numerator.

The method described in the next exercise is used whenever the whole number is not an exact divisor of the numerator.

2d Case: Multiplying the Denominator

103.

How much is
$\frac{3}{4} \div 2$?

Explanation.

$\frac{3}{4} \div 2 = \frac{1}{2}$ of $\frac{3}{4}$.

$$3\!/\!4 \div 2 = 3\!/\!8$$

EXERCISES

Divide (cancel whenever possible) :

1. $\frac{7}{8}$ by 3.	2. $\frac{5}{6}$ by 2.	3. $\frac{8}{9}$ by 3.
4. $\frac{7}{10}$ by 4.	5. $\frac{14}{15}$ by 5.	6. $\frac{9}{10}$ by 6.
7. $\frac{28}{39}$ by 14.	8. $\frac{12}{17}$ by 8.	9. $\frac{3}{37}$ by 12.
10. $\frac{21}{25}$ by 14.	11. $\frac{48}{55}$ by 36.	12. $\frac{81}{91}$ by 45.

For Reading and Study

The **reciprocal** of a whole number is a fraction whose denominator is the whole number and whose numerator is 1 ; *e.g.* the reciprocal of 5 is $\frac{1}{5}$; of 9 is $\frac{1}{9}$.

According to Art. 103 a fraction may be divided by a whole number by multiplying the fraction by the reciprocal of the whole number;

$$e.g. \ \tfrac{4}{5} \div 3 = \tfrac{1}{3} \text{ of } \tfrac{4}{5} = \tfrac{4}{15}.$$

From Arts. 101 and 103, we have:

RULE. **To divide a fraction by a whole number, divide the numerator of the fraction by the whole number, or multiply the fraction by the reciprocal of the whole number.**

Instead of actually multiplying, it is better to indicate the operation, and cancel wherever possible.

EXERCISES

104. Work mentally, using pencil only when the numbers are too large to be carried in the mind.

1. Divide by 10: $\frac{10}{25}$, $\frac{40}{41}$, $\frac{35}{36}$, $\frac{80}{90}$, $\frac{25}{27}$, $\frac{8}{9}$, $\frac{15}{16}$.

Find quotients:

2. $\frac{2}{3} \div 14$	**3.** $\frac{5}{8} \div 15$	**4.** $\frac{24}{25} \div 18$
5. $\frac{3}{4}$ mi. $\div 12$	**6.** $\frac{9}{10}$ yd. $\div 12$	**7.** $\frac{27}{31} \div 9$
8. $\frac{48}{49} \div 21$	**9.** $\frac{20}{30} \div 4$	**10.** $\frac{20}{21} \div 5$

In each of the following examples, reduce the mixed number to an improper fraction before dividing.

11. $3\frac{3}{4} \div 10$ **12.** $5\frac{5}{6} \div 20$ **13.** $6\frac{1}{4} \div 5$ **14.** $25\frac{1}{2} \div 3$
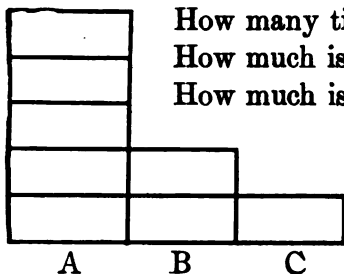
A Unit divided by a Fraction

FOR READING AND DISCUSSION

105. Looking at the picture, tell:

If A represents 1, what does C represent? B?
How many times is C contained in A?

How many times is B contained in A?

How much is $1 \div \frac{1}{5}$?

How much is $1 \div \frac{2}{5}$?

Looking at the picture we see that 2 fifths is contained in 1 unit or 5 fifths, $2\frac{1}{2}$ or $\frac{5}{2}$ times.

A B C

Therefore we have $1 \div \frac{2}{5} = \frac{5}{2}$.

Show by drawings the following:

$1 \div \frac{2}{7} = \frac{7}{2}$ $1 \div \frac{2}{9} = \frac{9}{2}$ $1 \div \frac{2}{3} = \frac{3}{2}$

$1 \div \frac{3}{5} = \frac{5}{3}$ $1 \div \frac{3}{4} = \frac{4}{3}$ $1 \div \frac{4}{9} = \frac{9}{4}$

When the numerator and denominator of a fraction exchange places, we say that the fraction is **inverted**. The new fraction is the **reciprocal** of the old fraction; *e.g.* $\frac{5}{2}$ is the reciprocal of $\frac{2}{5}$, $\frac{7}{2}$ is the reciprocal of $\frac{2}{7}$, $\frac{3}{1}$ or 3 is the reciprocal of $\frac{1}{3}$, etc.

We have proved the following:

PRINCIPLE. **1 divided by a fraction gives the reciprocal of the fraction.**

EXERCISES

Give quotients:

1. $1 \div \frac{5}{6}$	**2.** $1 \div \frac{4}{5}$	**3.** $1 \div \frac{5}{8}$
4. $1 \div \frac{1}{6}$	**5.** $1 \div \frac{3}{11}$	**6.** $1 \div \frac{7}{8}$
7. $1 \div \frac{9}{10}$	**8.** $1 \div \frac{5}{12}$	**9.** $1 \div \frac{5}{7}$

A Whole Number divided by a Fraction

FOR READING AND DISCUSSION

106. Compare $\begin{Bmatrix} 12 \div 3 = 4 \\ 24 \div 3 = 8 \end{Bmatrix}$ compare $\begin{Bmatrix} 1 \div 3 = \frac{1}{3} \\ 2 \div 3 = \frac{2}{3} \end{Bmatrix}$

What is the effect on the quotient, if the dividend be increased, while the divisor remains the same?

How much is $1 \div \frac{3}{4}$?

How much is $2 \div \frac{3}{4}$?

Explanation. $1 \div \frac{3}{4} = \frac{4}{3}$; $2 \div \frac{3}{4} = 2 \times \frac{4}{3} = \frac{8}{3}$, or $2\frac{2}{3}$.

Show that $4 \div \frac{3}{4} = 4 \times \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}$.

Show that $4 \div \frac{2}{5} = 4 \times \frac{5}{2} = 10$.

107. RULE. **To divide a whole number by a fraction, multiply the whole number by the reciprocal of the fraction.**

EXERCISES

·Find quotients by the short method; cancel whenever possible.

1. $6 \div \frac{3}{5}$ $10 \div \frac{5}{6}$ $9 \div \frac{2}{3}$ $12 \div \frac{4}{7}$ $8 \div \frac{5}{6}$

2. $10 \div \frac{3}{4}$ $9 \div \frac{4}{5}$ $15 \div \frac{5}{7}$ $7 \div \frac{1}{6}$ $12 \div \frac{4}{9}$

3. $8 \div 1\frac{1}{3}$ $7 \div 1\frac{3}{4}$ $9 \div 2\frac{1}{5}$ $12 \div 3\frac{3}{4}$ $20 \div 3\frac{1}{3}$
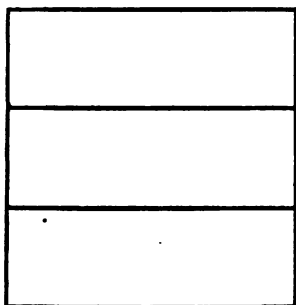
Give the explanation for Examples 4, 5, and 6.

4. $12 \div \frac{1}{4} = ?$ $12 \div \frac{3}{4} = ?$ $12 \div \frac{5}{4} = ?$

5. $18 \div 1\frac{1}{2} = ?$ $18 \div 2\frac{1}{4} = ?$ $18 \div 3\frac{1}{2} = ?$

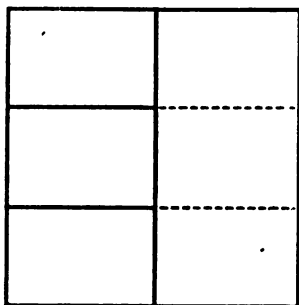6. $30 \div \frac{5}{6} = ?$ $75 \div 3\frac{5}{8} = ?$ $60 \div 4\frac{3}{4} = ?$

A Fraction divided by a Fraction

For Reading and Discussion

108. Compare $\begin{Bmatrix} 8 \div 2 = 4 \\ 4 \div 2 = 2 \end{Bmatrix}$ with $\begin{Bmatrix} 1 \div 3 = \frac{1}{3} \\ \frac{1}{2} \div 3 = \frac{1}{2} \text{ of } \frac{1}{3} = \frac{1}{6} \end{Bmatrix}$

$$1 \div 3 = \tfrac{1}{3} \qquad\qquad \tfrac{1}{2} \div 3 = \tfrac{1}{6}$$

What is the effect on the quotient if the dividend is decreased, while the divisor remains the same?

How many quarters of an inch in $\frac{1}{2}$ inch?

How much is $\frac{1}{2} \div \frac{1}{4}$?

How much is $1 \div \frac{1}{3}$?

How much is $\frac{1}{2} \div \frac{1}{3}$?

Explanation. $1 \div \frac{1}{3} = 3$; therefore $\frac{1}{2} \div \frac{1}{3} = \frac{1}{2}$ of $3 = \frac{3}{2}$ or $1\frac{1}{2}$.

How much is $1 \div \frac{3}{4}$?

How much is $\frac{2}{3} \div \frac{3}{4}$?

Explanation. $1 \div \frac{3}{4} = \frac{4}{3}$; $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3}$ of $\frac{4}{3} = \frac{8}{9}$.

Short Method. $\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$.

Rule. **To divide a fraction by a fraction, multiply the dividend by the reciprocal of the divisor.**

EXERCISES

109. Find quotients by the short method. Cancel whenever possible.

Solve mentally, using pencil only when numbers are too large to carry in the mind.

1. $\frac{3}{8} + \frac{2}{3}$	**6.** $\frac{5}{9} + \frac{3}{4}$	**11.** $\frac{7}{10} + \frac{2}{5}$	**16.** $\frac{5}{6} + \frac{9}{10}$
2. $\frac{7}{8} + \frac{2}{5}$	**7.** $\frac{4}{7} + \frac{4}{5}$	**12.** $\frac{3}{4} + \frac{2}{3}$	**17.** $3\frac{1}{3} + \frac{3}{4}$
3. $\frac{7}{8} + 1\frac{1}{3}$	**8.** $\frac{9}{10} + 1\frac{3}{4}$	**13.** $3\frac{3}{4} + 2\frac{2}{3}$	**18.** $7\frac{1}{2} + 1\frac{4}{5}$
4. $\frac{1}{2} + \frac{1}{4}$	**9.** $\frac{1}{2} + \frac{3}{4}$	**14.** $\frac{1}{2} + \frac{1}{8}$	**19.** $\frac{1}{2} + \frac{3}{8}$
5. $3\frac{1}{2} + 3\frac{1}{2}$	**10.** $5\frac{1}{3} + 1\frac{1}{6}$	**15.** $4\frac{3}{8} + 2\frac{1}{4}$	**20.** $10\frac{1}{3} + \frac{3}{10}$

Summary of Division of Fractions

FOR READING AND DISCUSSION

110. We have recognized three cases of division of fractions, as follows:

1. A fraction divided by a whole number.

E.g. $\frac{5}{8} + 3 = \frac{5}{8} \times \frac{1}{3} = \frac{5}{24}$.

2. A whole number divided by a fraction.

E.g. $5 + \frac{2}{3} = 5 \times \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$.

3. A fraction divided by a fraction.

E.g. $\frac{2}{5} + \frac{3}{8} = \frac{2}{5} \times \frac{8}{3} = \frac{16}{15} = 1\frac{1}{15}$.

NOTE. Since every mixed number may be reduced to an improper fraction, all examples containing mixed numbers are included in the three cases given.

Quote the rules given in the several cases from pages 102, 104, and 105.

These three cases which are solved by three different rules may be reduced to one case which is solved by one rule.

Looking at the 3 operations, we see that in each case the dividend (or number before the division sign) is multiplied by the reciprocal of the divisor. Hence we have *one* rule for all cases of division of fractions:

RULE. **Multiply the dividend by the reciprocal of the divisor.**

Another way to state the rule is:

RULE. **Invert the divisor and multiply.**

You can see how important it is to know which number is the divisor. If you should make the mistake of inverting the dividend, your answer would be wrong.

ORAL EXERCISES

111. Give quotients:

1. $\frac{1}{3} \div 6$	6. $\frac{2}{3} \div 6$	11. $8 \div \frac{1}{2}$
2. $\frac{1}{5} \div 5$	7. $\frac{1}{8} \div 3$	12. $14 \div \frac{1}{4}$
3. $\frac{2}{7} \div 4$	8. $\frac{3}{8} \div 9$	13. $24 \div \frac{1}{3}$
4. $\frac{3}{4} \div 12$	9. $\frac{5}{6} \div 10$	14. $16 \div \frac{2}{3}$
5. $\frac{4}{9} \div 8$	10. $\frac{1}{6} \div 5$	15. $20 \div \frac{4}{5}$

16. $12 + \frac{1}{6}$	21. $\frac{3}{4} + \frac{1}{2}$	26. $\frac{5}{7} + \frac{1}{2}$
17. $28 + \frac{2}{7}$	22. $\frac{5}{8} + \frac{1}{8}$	27. $\frac{2}{9} + \frac{1}{4}$
18. $32 + \frac{8}{9}$	23. $\frac{3}{5} + \frac{1}{3}$	28. $\frac{3}{10} + \frac{2}{3}$
19. $48 + \frac{3}{4}$	24. $\frac{5}{6} + \frac{1}{6}$	29. $\frac{3}{8} + \frac{5}{12}$
20. $36 + \frac{6}{4}$	25. $\frac{7}{8} + \frac{3}{4}$	30. $\frac{9}{10} + \frac{3}{5}$

EXERCISES

112. Give quotients:

1. $\frac{1}{2} + 1\frac{1}{2}$	11. $1\frac{1}{8} + \frac{3}{4}$	21. $1\frac{1}{4} + 1\frac{1}{4}$
2. $\frac{1}{4} + 1\frac{1}{4}$	12. $2\frac{1}{3} + \frac{2}{7}$	22. $1\frac{1}{2} + 1\frac{1}{5}$
3. $\frac{1}{6} + 1\frac{1}{3}$	13. $5\frac{1}{2} + \frac{5}{6}$	23. $1\frac{3}{4} + 2\frac{1}{4}$
4. $\frac{1}{9} + 2\frac{1}{4}$	14. $1\frac{1}{4} + \frac{3}{5}$	24. $2\frac{1}{2} + 1\frac{2}{3}$
5. $\frac{3}{4} + 2\frac{1}{2}$	15. $2\frac{1}{2} + \frac{1}{10}$	25. $2\frac{3}{4} + 1\frac{3}{4}$
6. $\frac{5}{6} + 1\frac{1}{4}$	16. $4\frac{1}{2} + \frac{9}{10}$	26. $2\frac{1}{3} + 4\frac{2}{3}$
7. $\frac{3}{8} + 2\frac{1}{4}$	17. $3\frac{1}{3} + \frac{7}{10}$	27. $3\frac{1}{2} + 2\frac{1}{2}$
8. $\frac{7}{8} + 2\frac{1}{2}$	18. $4\frac{1}{2} + \frac{3}{8}$	28. $1\frac{1}{8} + 1\frac{3}{8}$
9. $\frac{4}{5} + 1\frac{4}{5}$	19. $3\frac{1}{8} + \frac{5}{6}$	29. $1\frac{7}{8} + 2\frac{1}{4}$
10. $\frac{2}{3} + 3\frac{1}{3}$	20. $4\frac{3}{4} + \frac{7}{8}$	30. $2\frac{1}{5} + 2\frac{1}{2}$

WRITTEN EXERCISES

113. Find quotients:

1. $35 + \frac{5}{9}$	4. $45 + \frac{9}{13}$	7. $144 + 1\frac{2}{19}$
2. $29 \div \frac{2}{11}$	5. $68 \div 1\frac{2}{17}$	8. $50 + \frac{3}{10}$
3. $36 + \frac{8}{9}$	6. $132 + \frac{9}{11}$	9. $35 + \frac{4}{5}$

10. $28 + \frac{5}{9}$ 17. $\frac{9}{196} + 45$ 24. $3\frac{8}{9} + 2\frac{1}{6}$

11. $\frac{9}{20} + 72$ 18. $\frac{121}{200} + 44$ 25. $4\frac{7}{10} + 3\frac{2}{5}$

12. $\frac{11}{30} + 88$ 19. $\frac{17}{100} + 69$ 26. $6\frac{3}{8} + 7\frac{5}{8}$

13. $\frac{19}{50} + 47$ 20. $\frac{8}{75} + 48$ 27. $15\frac{2}{3} + 1\frac{7}{9}$

14. $\frac{14}{25} + 70$ 21. $5\frac{3}{8} + 2\frac{3}{4}$ 28. $26\frac{1}{2} + 13\frac{1}{4}$

15. $\frac{29}{60} + 240$ 22. $7\frac{1}{2} + 2\frac{1}{7}$ 29. $17\frac{1}{4} + 5\frac{3}{4}$

16. $\frac{141}{365} + 35$ 23. $12\frac{1}{2} + 3\frac{1}{3}$ 30. $23\frac{5}{8} + 4\frac{29}{40}$

WRITTEN EXERCISES

MISCELLANEOUS

114. Find the value of:

1. $\frac{3}{8} - \frac{1}{4} + 4\frac{3}{5}$ 13. $\frac{3}{5}$ of $\frac{3}{7}$ of \$350

2. $\frac{3}{8} \times \frac{5}{9} \times \frac{8}{15}$ 14. $18\frac{3}{8} + 2\frac{5}{6} + 1\frac{1}{3} - \frac{8}{9}$

3. $12\frac{5}{6} + 2\frac{1}{3}$ 15. $\frac{5}{9}$ of $\frac{2}{3}$ of 540

4. $6\frac{1}{4} \times 5\frac{1}{2} \times 8$ 16. $\frac{7}{8} \times 3\frac{1}{3} \times 5\frac{1}{4}$

5. $\frac{5}{9} \div 13\frac{1}{2}$ 17. $\frac{6}{7} \times \frac{5}{9} \times 10\frac{1}{2}$

6. $9\frac{1}{4} \times 3\frac{1}{2} \times 20$ 18. $18\frac{1}{3} \div (\frac{5}{6} \times \frac{1}{4})$

7. $\frac{7}{9} + 2\frac{3}{5} + 6\frac{1}{2} - \frac{1}{3}$ 19. $(27 + 12\frac{1}{2}) \times \frac{3}{8}$

8. $\frac{2}{5} \times \frac{4}{9} \div \frac{3}{5}$ 20. $\frac{6}{7} + \frac{7}{10} + \frac{3}{5} - \frac{9}{10}$

9. $\frac{3}{5}$ of $(2\frac{1}{2} + 3\frac{1}{4} + \frac{7}{8})$ 21. $\frac{1}{2}$ of $\frac{3}{4}$ of 270

10. $5\frac{1}{4} \times 2\frac{1}{2} + \frac{7}{12}$ 22. $35\frac{4}{7} + 23\frac{1}{3}$

11. $2\frac{1}{2} \times 3\frac{1}{3} \times \frac{7}{12}$ 23. $6\frac{1}{2} \times 8\frac{3}{5} \times \frac{9}{10}$

12. $4\frac{1}{3} + 9\frac{3}{5} - 2\frac{1}{4}$ 24. $18\frac{1}{4} \times 4 \div \frac{7}{9}$

25. $\frac{3}{8} + \frac{4}{9} - \frac{7}{12} + \frac{5}{6}$
26. $\frac{6}{7}$ of $(19\frac{3}{8} - 4\frac{1}{8})$
27. $25\frac{3}{8} + 16\frac{1}{4}$

28. $5\frac{3}{4} - \frac{3}{5} - \frac{3}{8} - \frac{1}{2}$
29. $16\frac{1}{4} + 5\frac{1}{8}$
30. $\frac{9}{10}$ of $\frac{3}{8}$ of \$500

WRITTEN EXERCISES

115. Find the missing numbers:

1. $8 + ? = 32$
2. $15 + ? = 45$
3. $20 + ? = 100$
4. $25 + ? = 75$
5. $16 \div ? = 10$
6. $24 + ? = 32$
7. $9 + ? = 15$
8. $36 + ? = 42$
9. $27 \div ? = 30$
10. $30 + ? = 27$

11. $2\frac{5}{9} + \frac{5}{8} = ?$
12. $3\frac{4}{5} + \frac{7}{12} = ?$
13. $7\frac{1}{3} + \frac{3}{5} = ?$
14. $9\frac{2}{7} + 8\frac{1}{2} = ?$
15. $5\frac{3}{10} + 4\frac{3}{5} = ?$
16. $8\frac{3}{4} + 6\frac{1}{4} = ?$
17. $2\frac{5}{9} + 3\frac{2}{3} = ?$
18. $3\frac{7}{12} + 6\frac{1}{4} = ?$
19. $6\frac{1}{4} + 5\frac{1}{3} = ?$
20. $8\frac{2}{3} + 2\frac{1}{4} = ?$

WRITTEN EXERCISES

116. Find the missing numbers:

1. $? + \frac{3}{10} = \frac{5}{8}$
2. $? + \frac{4}{9} = \frac{3}{5}$
3. $? + \frac{7}{15} = \frac{2}{9}$
4. $? + \frac{19}{20} = \frac{3}{8}$

5. $? + \frac{21}{25} = \frac{4}{5}$
6. $? + \frac{23}{30} = \frac{9}{10}$
7. $? + \frac{10}{21} = \frac{5}{6}$
8. $? \div \frac{9}{14} = \frac{3}{7}$

9. $? \div \frac{12}{23} = \frac{14}{15}$

10. $? \div \frac{10}{21} = \frac{7}{25}$

11. $? \div 3\frac{1}{3} = 2\frac{1}{8}$

12. $? \div 2\frac{4}{5} = 3\frac{4}{9}$

13. $? \div 3\frac{5}{7} = 5$

14. $? \div 2\frac{3}{4} = 6\frac{1}{4}$

15. $? \div 5\frac{3}{8} = 3\frac{3}{8}$

16. $? \div 6\frac{3}{5} = 1\frac{1}{9}$
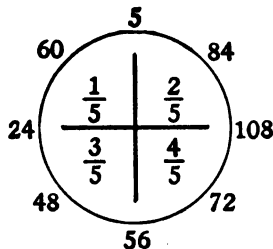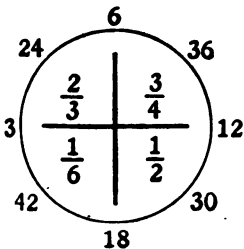
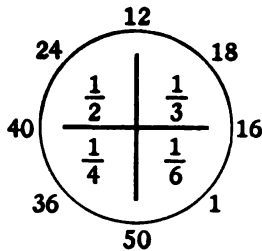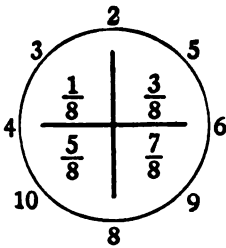17. $? \div 4\frac{2}{9} = 2\frac{3}{4}$

18. $? \div 10\frac{3}{5} = 7\frac{1}{2}$

19. $? \div 15\frac{1}{2} = 6$

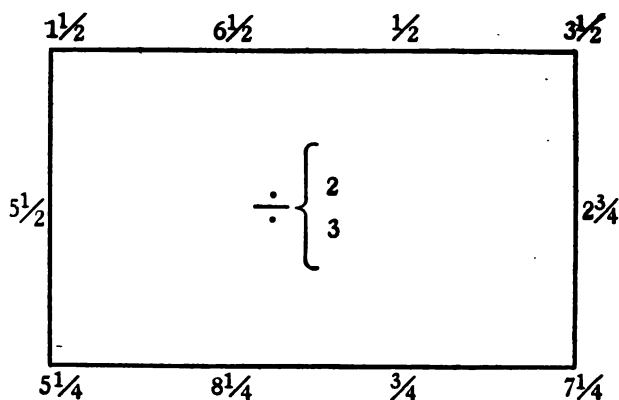20. $? \div 12\frac{9}{12} = 3\frac{1}{4}$

SIGHT DRILL

117. Divide the numbers on the circumference by each fraction in turn.

SIGHT DRILL

118. Give quotients rapidly:

5	25	50

$$1 \quad \div \begin{cases} 1\frac{1}{4} \\ 2\frac{1}{2} \end{cases} \quad 15$$

75	10	100

$1\frac{1}{2}$	$6\frac{1}{2}$	$\frac{1}{2}$	$3\frac{1}{2}$

$$5\frac{1}{2} \quad \div \begin{cases} 2 \\ 3 \end{cases} \quad 2\frac{3}{4}$$

$5\frac{1}{4}$	$8\frac{1}{4}$	$\frac{3}{4}$	$7\frac{1}{4}$

NOTE TO TEACHER. Various forms of "Fixing Devices," such as those in Book 1, pp. 73, 74, 75, 76, 124, 125, 126, etc., are as valuable for daily drills in fractions, as in fundamental operations with integers.

| 36 |
| 21 |
| 42 |
| 30 |
| 9 |
| 3 |
| 24 |
| 6 |
| 15 |
| 27 |
| 12 |
| 18 |

$\div \frac{3}{4}$

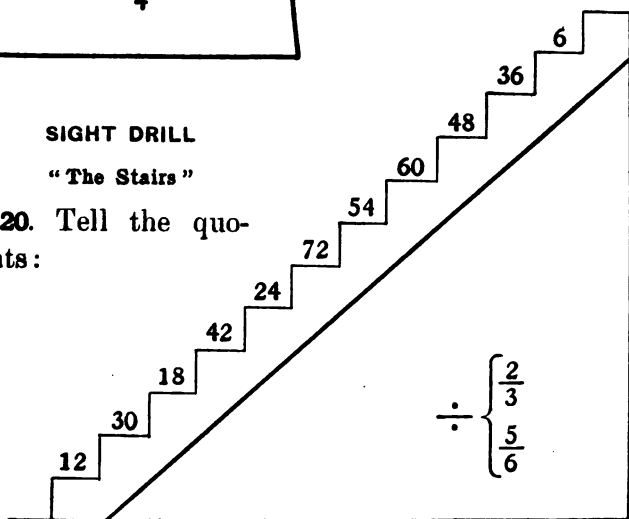### SIGHT DRILL

"The Ladder"

119. **1.** Begin at the lowest step; go up as quickly as possible.

2. Begin at the highest step; go down as quickly as possible.

3. Make other ladders, using other numbers.

SIGHT DRILL

"The Stairs"

120. Tell the quotients:

6

36

48

60

54

72

24

42

18

30

12

$\div \begin{cases} \dfrac{2}{3} \\ \dfrac{5}{6} \end{cases}$

I

121. 1. How many pints are there in 4 qt. ?

2. How many quarters are there in $6 ?

3. How many strips of paper each $\frac{3}{4}$ ft. wide are needed to cover a wall 8 ft. long ?

4. How many strips of paper each 1$\frac{1}{2}$ ft. wide are needed to cover a wall 12 ft. long ?

5. A pitcher contains 3 qt. of milk. How many pint bottles can be filled from it ?

6. How many packages, each containing $\frac{1}{4}$ lb. of pepper, can be filled from a package containing 8 lb. ?

7. 14 lb. of sugar are put up in 3$\frac{1}{2}$ lb. packages. How many packages are there ?

8. A board 4 ft. wide is cut into strips. Each strip is $\frac{1}{8}$ ft. wide. How many strips are obtained?

9. How many steps each 2$\frac{1}{2}$ ft. long are taken in walking 15 ft. ?

10. 24 doz. lead pencils are packed in boxes. Each box contains $\frac{1}{4}$ doz. How many boxes are there ? What is the total cost at 10 ct. a box ?

11. $\frac{3}{8}$ of a pound of flour is used in making a loaf of bread. How many loaves can be made from 18 lb. of flour?

12. If a man earns $\frac{3}{5}$ an hour, how long will it take him to earn 5\frac{1}{4}$?

13. If a man earns $8½ by working 3⅔ hr., how much does he earn in one hour?

14. A horse runs 2½ mi. an hour. How long will it take the horse to run 12¼ mi.?

15. A horse runs 6¾ mi. in 3 hr. How many miles will he run in 1 hr.?

16. A piece of ribbon 9¾ yd. long is cut into 6 equal pieces. How long is each piece?

17. A piece of lace 10½ yd. long is cut into pieces. Each piece is 1½ yd. long. How many pieces are there?

18. A piece of picture molding 25½ ft. long is cut into 4 equal pieces. How long is each piece?

19. 48 qt. of berries are to be put into baskets, each basket containing 1½ qt. How many baskets are needed?

20. ½ lb. of coffee costs 17 ct. Find the cost of ¼ lb.

21. ½ lb. of coffee costs $⅕. How many pounds may be bought for $⅘?

22. How much is gained by buying 144 buttons at 3 for 1 ct. and selling them at 2 for 1 ct.?

23. Books cost $⅔ each. How many may be purchased for $10? For $18? For $24?

24. Leather belting costs $⅗ a foot. How many feet may be purchased for $9? For $12? For $30?

25. $3\frac{1}{2}$ yd. of cloth costs $\$5\frac{1}{4}$. Find the cost of 1 yd.

26. $2\frac{1}{2}$ yd. of linen costs $\$1\frac{1}{4}$. Find the cost of 1 yd.

27. $3\frac{1}{2}$ yd. of cloth costs $\$5\frac{3}{4}$. How many yards may be bought for $\$28\frac{1}{2}$?

28. $2\frac{1}{2}$ yd. of linen costs $\$1\frac{3}{4}$. How many yards may be bought for $\$15\frac{1}{4}$?

29. $2\frac{3}{4}$ yd. of lace costs $\$6\frac{1}{20}$. Find the cost of 1 yd.

30. A boy earned $\$36$. He earned $\$4\frac{1}{2}$ in 1 week. How many weeks did he work?

31. A man earns $\$15\frac{3}{4}$ a week and spends $\$7\frac{1}{4}$ each week. In how many weeks will he save $\$51$?

32. Pencils are bought at $1\frac{1}{4}$ ct. each and sold at $2\frac{1}{2}$ ct. each. How many must be sold to make a profit of 25 ct.?

COMPLEX FRACTIONS

FOR READING AND STUDY

122. A **complex fraction** is a fraction whose numerator or denominator contains a fraction; *e.g.* $\frac{\frac{1}{2}}{5}$, $\frac{3}{\frac{2}{3}}$, $\frac{2\frac{1}{2}}{5\frac{2}{5}}$.

A complex fraction is read as an indicated division; *e.g.* $\frac{\frac{1}{2}}{5}$ is read one half divided by 5; $\frac{3}{\frac{2}{3}}$ is

read 3 divided by $\frac{3}{4}$, etc. (See Art. 50. A fraction is an indicated division.)

A complex fraction may be reduced to a simple fraction by performing the operations indicated.

123. Reduce $\frac{\frac{1}{2}}{5}$ to a simple fraction.

$$\frac{\frac{1}{2}}{5} = \frac{1}{2} \div 5 = \frac{1}{10} \quad Ans.$$

Simplify $\frac{2\frac{1}{2}}{5\frac{2}{5}}$.

$$\frac{2\frac{1}{2}}{5\frac{2}{5}} = 2\frac{1}{2} \div 5\frac{2}{5} = \frac{5}{2} \times \frac{5}{27} = \frac{25}{54} \quad Ans.$$

Simplify : 1. $\frac{2\frac{1}{4}}{3\frac{1}{3}}$ 2. $\frac{8}{\frac{5}{6}}$ 3. $\frac{\frac{3}{4}}{15}$ 4. $\frac{\frac{9}{10}}{12}$ 5. $\frac{3\frac{3}{4}}{8\frac{1}{3}}$

SHORT METHOD. If the denominator of a complex fraction is an integer, the fraction can usually be reduced at sight to a simple fraction by multiplying both numerator and denominator by the denominator of the fraction in the numerator.

Simplify $\frac{\frac{2}{3}}{5}$. Multiplying both terms by 3, we have $\frac{\frac{2}{3} \times 3}{5 \times 3} = \frac{2}{15}$.

Simplify $\frac{2\frac{5}{6}}{15}$.

We say: $6 \times 2 = 12$; $12 + 5 = 17$; $6 \times 15 = 90$. The answer is $\frac{17}{90}$.

124. Reduce to simple fractions:

1. $\dfrac{4\frac{1}{2}}{6}$ 2. $\dfrac{3\frac{1}{3}}{10}$ 3. $\dfrac{2\frac{2}{5}}{3}$ 4. $\dfrac{8\frac{1}{3}}{10}$ 5. $\dfrac{2\frac{2}{3}}{5}$

125. Reduce to the simplest form:

1. $\dfrac{\frac{3}{8}}{12}$ 2. $\dfrac{\frac{4}{7}}{\frac{2}{3}}$ 3. $\dfrac{4\frac{1}{3}}{5\frac{3}{5}}$ 4. $\dfrac{\frac{3}{4}}{12\frac{1}{2}}$ 5. $\dfrac{2\frac{1}{2}}{100}$

A Short Method of dividing a Mixed Number by a Whole Number

126. 1. Divide $83\frac{1}{3}$ by 5.

Process

$$16\frac{3\frac{1}{3}}{5}$$
$$5)\overline{83\frac{1}{3}} = 16\frac{10}{15} = 16\frac{2}{3}$$

Explanation. Dividing $83\frac{1}{3}$ by 5, we obtain 16 for a quotient and $3\frac{1}{3}$ for a remainder; reducing $\dfrac{3\frac{1}{3}}{5}$ to a simple fraction, our answer is $16\frac{2}{3}$.

2. Divide $427\frac{3}{4}$ by 5; by 6; by 50; by 7.

Find quotients:

3. $256\frac{5}{8} + 12$ 4. $808\frac{5}{9} + 20$

5. $125\frac{4}{7} + 10$ 6. $875\frac{5}{16} + 32$

TYPE PROBLEMS INVOLVING FRACTIONAL RELATIONS OF NUMBERS

127. All problems involving but one operation in the multiplication and division of fractions may be reduced to one of the three following cases.

First Type Problem
Finding a fractional part of a number

128. 1. If flour is selling at $9 a barrel, how much should I pay for $\frac{2}{3}$ of a barrel?

Explanation. $\frac{2}{3}$ of a barrel costs $\frac{2}{3}$ of the price of a whole barrel; I should pay $\frac{2}{3}$ of $9, which is $6.

ORAL EXERCISES

2. How many pounds in $\frac{3}{16}$ of a ton?

3. How much is $\frac{5}{9}$ of 50? Of 80? Of $6\frac{2}{3}$? Of $4\frac{1}{5}$?

4. Find $\frac{7}{10}$ of $160; $\frac{5}{12}$ of $96; $\frac{8}{9}$ of 300 yd.

5. Find $\frac{1}{8}$ of 240 yd. Of 320 yd. Of 640 yd.

6. How much is $\frac{5}{8}$ of 40 ft.? Of 120 ft.? Of 360 ft.?

7. How much greater is $\frac{3}{10}$ of 70 than $\frac{1}{8}$ of 40?

8. How much less is $\frac{3}{8}$ of 80 than $\frac{3}{5}$ of 100?

9. How much is $\frac{2}{5}$ of $2000? $\frac{4}{5}$ of $2000?

Second Type Problem

Finding what fraction one number is of another number

129. What part of 9 is 1?

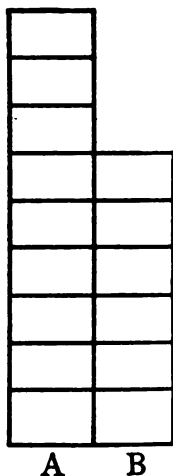What part of 9 is 6?

What part of 9 is 5?

What part of 9 is 4?

What part of 9 is 7?

What part of 9 is 8?

What part of 9 is 2?

What part of 9 is 3?

What part of 9 is 9?

A   B

ORAL PROBLEMS

1. John had 30 ct.; he spent 1 ct. What part of his money did he spend?

2. John had 30 ct.; he spent 10 ct. What part of his money did he spend?

Process. 10 ct. = $\frac{10}{30}$ or $\frac{1}{3}$ of 30 ct. The answer is $\frac{1}{3}$.
Or 10 ct. ÷ 30 ct. = $\frac{10}{30}$ = $\frac{1}{3}$ *Ans.*

3. A pole 24 ft. long has 10 ft. of its length in water. What part of the pole is in water? Out of water?

4. What part of a bushel is 20 qt.?

Test your results in the following examples in this way:
Suppose that your answer to Ex. 5 is $\frac{1}{3}$. Test: $\frac{1}{3}$ of 60 = 20.
$\frac{1}{3}$ is right.

What part of

    5. 60 is 20?   **6.** 50 is 30?

    7. 75 is 50?   **8.** 45 is 25?

9. A grocer sold 21 gal. of vinegar from a cask of 35 gal. What part of the cask was sold? What part was left?

10. A clerk's weekly expenses are $25; his weekly salary is $30. What part of his salary may he save?

11. A man's account in the savings' bank was increased in one year from $500 to $600. The increase in savings was what fractional part of the amount of his savings at the beginning of the year?

12. A man sold for $300 a horse which cost $250. What part of the cost was the gain?

13. If 18 yd. of silk cost $21, what will 12 yd. cost?

Process. 12 yd. will cost $\frac{12}{18}$ or $\frac{2}{3}$ of $21, or $14.

14. If 24 pairs of shoes cost $40, what will 18 pairs cost?

15. If 32 yd. of cloth are worth $5, how many yards can be bought for $15?

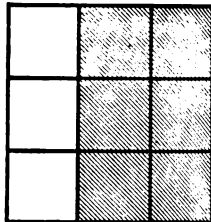**16.** If $48 is the value of 18 boxes of lemons, how many boxes can be bought for $40?

17. Make five original problems of the second type.

18. Make five original problems of the first type.

Third Type Problem

Finding the whole when a fractional part is given

First Case

130. 1. 6 is $\frac{2}{3}$ of what number?

$\frac{1}{3}$ is what part of $\frac{2}{3}$?

$\frac{1}{2}$ of 6 is what part of 9?

Show that $\frac{1}{2}$ of 6 is equal to $\frac{1}{3}$ of the whole figure.

If $\frac{2}{3}$ represents $\$12$, what does $\frac{3}{3}$ represent?

12 is $\frac{2}{3}$ of what number?

2. What is the cost of a yard of ribbon if $\frac{3}{4}$ yd. costs 30 ct.?

Process

$\frac{3}{4}$ yd. costs 30 ct.

$\frac{1}{4}$ yd. costs $\frac{1}{3}$ of 30 ct. $= 10$ ct.

$\frac{4}{4}$ yd. costs 4×10 ct. $= 40$ ct.

Or 30 ct. $\div \frac{3}{4} = \overset{10}{\cancel{30}} \times \dfrac{4}{\cancel{3}} = 40$ ct.

Explanation. The cost of $\frac{1}{4}$ of a yard is $\frac{1}{3}$ of the cost of $\frac{3}{4}$ of a yard. Therefore the cost of $\frac{1}{4}$ of a yard is $\frac{1}{3}$ of 30 ct., or 10 ct.

The cost of 1 yd. is 4 times the cost of $\frac{1}{4}$ of a yard. Therefore the cost of 1 yd. is 4 times 10 ct., or 40 ct.

Shorter Form of Explanation. The cost of 1 yd. is $\frac{4}{3}$ of the cost of $\frac{3}{4}$ of a yard. Therefore, the cost of 1 yd. is $\frac{4}{3}$ of 30 ct., or 40 ct.

3. If $\frac{5}{8}$ of a cord of wood costs $\$10$, what will a cord cost?

Test. My answer is 16. Does $\frac{5}{8}$ of 16 equal 10? Yes. Then 16 is right.

4. If $\frac{4}{5}$ of a line measures 24 in., how long is $\frac{1}{5}$ of the line? How long is the whole line?

5. $18 is $\frac{3}{10}$ of what amount?

6. 12 is $\frac{1}{5}$ of what number?

7. 12 is $\frac{2}{5}$ of what number?

8. 12 is $\frac{3}{5}$ of what number?

9. 12 is $\frac{4}{5}$ of what number?

10. 27 is $\frac{3}{8}$ of what number?

11. 15 is $\frac{5}{6}$ of what number?

12. 25 ct. is $\frac{1}{3}$ of what amount?

13. 20 ft. is $\frac{2}{5}$ of how many feet?

14. 24 books is $\frac{3}{5}$ of how many books?

15. 50 lb. is $\frac{5}{6}$ of the weight of a bushel of wheat. How much is the weight of a bushel?

16. Give three original problems of the third type.

Second Case

For Reading and Discussion
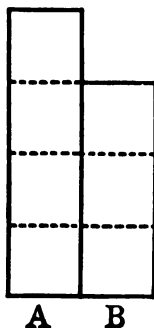
131. If A is 1, what is B?

If A represents $\frac{1}{4}$, what does B represent?

If B is 3, what is A?

What is A, if B is 6? If B is 12?

What does A equal if B equals 9 lb.?

What is A worth if B is worth 15 ct.?

PROBLEMS

1. A boy cut off $\frac{1}{4}$ of his top cord, leaving it 6 ft. long. How long was it at first?

Process

$6 \div \frac{3}{4} = 6 \times \frac{4}{3} = 8$

The answer is 8 ft.

Explanation. Since $\frac{1}{4}$ of the top cord was cut off, 6 ft. is $\frac{3}{4}$ of its original length.

$\frac{3}{4}$ of the original length $= 6$ ft.
$\frac{1}{4}$ of the original length $= \frac{1}{3}$ of 6 ft. $= 2$ ft.
$\frac{4}{4}$ of the original length $= 4 \times 2$ ft. $= 8$ ft.

Shorter Form of Explanation. Since $\frac{1}{4}$ of the top cord was cut off, 6 ft. is $\frac{3}{4}$ of the original length. Hence the original length is $\frac{4}{3}$ of 6 ft. or 8 ft.

2. A girl spends $\frac{1}{4}$ of her weekly pocket money and has 15 ct. left. How much is her weekly pocket money?

3. A wood cutter cuts off $\frac{1}{5}$ of a log, leaving it 12 ft. long. How long was it before cutting?

4. A piece of cloth shrank $\frac{1}{10}$ of its length, leaving it 45 yd. long. How long was it before shrinking?

5. A teacher throws away $\frac{1}{8}$ of the spelling books as unfit for use; there are 35 left. How many were there at first?

6. In a certain class $\frac{2}{5}$ of the pupils were absent; 24 were present. How many pupils in the entire class?

7. What number decreased by $\frac{1}{6}$ of itself equals 20?

8. What number decreased by $\frac{1}{4}$ of itself equals 18?

9. What number minus $\frac{1}{6}$ of itself equals 25?

10. A number decreased by $\frac{2}{5}$ of itself equals 12. Find the number.

11. A number decreased by $\frac{5}{8}$ of itself equals 24. Find the number.

12. Alice has 50 ct.; this is $\frac{1}{3}$ less than the amount that Jane has. How much has Jane?

13. A man walked 6 mi. and still had $\frac{3}{4}$ of his journey to travel. How many miles in his entire journey?

14. $\frac{9}{10}$ of a class was promoted; 4 pupils were not promoted. How many were in the class? How many were promoted?

15. $\frac{1}{15}$ of a class was absent; 42 children were present. How many children were absent?

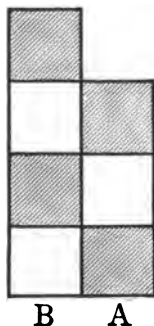**16.** Make up three examples similar to the 12th, 13th, and 15th.

Third Case

132. If A is 1, what is B?

A represents $\frac{3}{5}$. How many thirds does B represent?

If B is 8, what is A?

What is A, if B is 12? If B is 16?

B   A

What does A equal, if B equals 20 lb.?

What is A worth, if B is worth 40 ct.?

A boy adds to his fishing line $\frac{1}{3}$ of its original length, making it 12 ft. long. How long was it at first?

Short Method

$$12 \text{ ft.} \div \frac{4}{3} = 12 \text{ ft} \times \frac{3}{4}$$
$$= 9 \text{ ft.}$$

The answer is 9 ft.

Explanation. Since $\frac{1}{3}$ was added to the original $\frac{3}{3}$, the line is $\frac{4}{3}$ of the original length.

$\frac{4}{3}$ of the original length = 12 ft.

$\frac{1}{3}$ of the original length = $\frac{1}{4}$ of 12 ft. = 3 ft.

$\frac{3}{3}$ of the original length = 3×3 ft. = 9 ft. *Ans.*

Shorter Form of Explanation. Since $\frac{1}{3}$ was added to the original length, 12 ft. is $\frac{4}{3}$ of the original length. Hence the original length is $\frac{3}{4}$ of 12 ft. or 9 ft.

PROBLEMS

Solve mentally, using pencil only when the numbers are too large to be carried in the mind:

1. The weight of a toy and its box is 10 lb. The box weighs $\frac{1}{4}$ as much as the toy. Find the weight of each.

2. Harry and John together weigh 180 lb. John weighs $\frac{1}{2}$ as much as Harry. Find the weight of each boy.

3. A clerk's weekly wages are increased by $\frac{1}{6}$ of the former amount. He now receives $14 a week. How much did he receive at first?

4. A piece of fur is stretched so that its length is increased by $\frac{1}{5}$ of its former length. If its length after stretching is 3 ft., what was its length at first?

5. Some new pupils were admitted to a class, making the register 55. This was $\frac{11}{10}$ of the former register. How many pupils were admitted?

6. Make up three problems similar to the 3d, 4th, and 5th.

7. The number of men employed in a factory was increased by $\frac{1}{7}$, making the total number 128. How many men were employed at first?

8. A landlord increased the rent by $\frac{1}{9}$ of the original amount, making the new rent $400 a month. What was the original rent?

9. A man sold a watch at a gain of $\frac{1}{5}$ of the original cost. He sold it for $60. Find the cost. Find the gain.

MISCELLANEOUS PROBLEMS IN FRACTIONS

ORAL PROBLEMS

133. 1. One book weighs $\frac{3}{8}$ lb.; another $\frac{3}{4}$ lb. What is the weight of the two books?

2. A box contains a number of toys. The toys weigh $10\frac{1}{2}$ lb.; the box weighs $1\frac{1}{4}$ lb. Find the total weight of the toys and the box.

NOTE. See the Forms of Explanation on pages 25–27. The forms of analysis are the same for fractions as for integers.

3. Three baskets weigh $\frac{3}{8}$ lb., $\frac{5}{8}$ lb., and $\frac{7}{8}$ lb., respectively. Find the total weight.

4. Harry receives $ $5\frac{3}{4}$ a week. George receives $ $4\frac{1}{2}$ a week. How much do both receive in a week?

5. How much more does Harry receive than George?

6. A box filled with coal weighs $20\frac{7}{8}$ lb. The box when empty weighs $1\frac{1}{4}$ lb. What is the weight of the coal?

7. Nellie has two pieces of ribbon. One piece is $8\frac{1}{2}$ yd. long; the other is $4\frac{1}{4}$ yd. long. What is the total length of both pieces?

8. How much longer is one piece than the other?

9. A girl spends $\frac{3}{4}$ of an hour studying arithmetic, $\frac{1}{2}$ an hour studying spelling, and $\frac{1}{3}$ of an hour making a drawing. How much time does she spend at her lessons?

10. How much more time does she spend on the arithmetic lesson than on the drawing?

11. A milkman sold $2\frac{1}{2}$ qt., $3\frac{1}{2}$ qt., 5 qt., and $7\frac{1}{2}$ qt. How much milk did he sell?

12. One piece of rope is $10\frac{3}{8}$ ft. long; a second piece of rope is $8\frac{1}{2}$ ft. long. How much longer is the first piece than the second?

13. An express train travels at the rate of $40\frac{1}{4}$ mi. an hour; a slower train travels at the rate of $25\frac{3}{4}$ mi. an hour. How much farther does the first train go in an hour than the second train?

14. The receipts of a store were: $\$10\frac{1}{2}$, $\$15\frac{1}{4}$, $\$21\frac{3}{4}$, $\$16$. Find the total receipts.

15. Sugar was bought at an average of $3\frac{7}{8}$ ct. a pound and sold at an average of $5\frac{1}{2}$ ct. a pound. Find the profit on 1 lb. On 5 lb.

16. If $2\frac{1}{4}$ ct. profit is made on a blank book, how much profit will be made on 48 books?

17. A boy walked the following distances: $2\frac{1}{8}$ mi., $2\frac{1}{3}$ mi., $3\frac{1}{2}$ mi.; how far did he walk?

18. A farmer sells $12\frac{1}{2}$ doz. eggs, $6\frac{1}{4}$ doz., $7\frac{2}{3}$ doz., and $8\frac{1}{6}$ doz. How many dozen in all are sold? How many eggs?

19. A boat sails $10\frac{5}{8}$ mi., $12\frac{1}{6}$ mi., $11\frac{2}{3}$ mi., and $10\frac{1}{2}$ mi. Find the total distance.

20. If a man earns $\$30\frac{3}{4}$ a week and spends $\$15\frac{1}{2}$ a week, how much does he save in 3 weeks?

21. A can contained 40 qt. of milk. If $12\frac{1}{2}$ qt. were sold, how many quarts were left?

22. A piece of wood $5\frac{3}{8}$ ft. long was cut from a board 10 ft. long. How much of the wood was left?

23. How many pints are there in $8\frac{1}{2}$ qt.?

24. How many feet are there in $7\frac{1}{3}$ yd.?

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25. Find the cost of $3\frac{1}{2}$ lb. of sugar at 6 ct. a pound.

26. A basket holding $6\frac{1}{2}$ bu. of wheat was filled from a bin containing 30 bu. How many bushels were left in the bin?

27. William had $\$10\frac{1}{2}$. He spent $\$2\frac{1}{2}$ for gloves and $\$1\frac{1}{4}$ for a cap. How much money did he have left?

28. How many quarter dollars in $\$12\frac{1}{2}$?

29. How many quarter dollars in $\$15\frac{3}{4}$?

30. A line 21 yd. long is measured by a rule which is $\frac{1}{3}$ yd. long. How many times must the rule be used to measure the line?

31. How many pint bottles can be filled from 8 qt. of milk?

32. How many pint bottles can be filled from 4 gal. of milk? (8 pt. = 1 gal.)

33. How many $\frac{1}{2}$-pt. bottles can be filled from 6 gal. of milk?

34. There are 24 hours in a day. If a boy sleeps $9\frac{2}{3}$ hr., how many hours is he awake?

35. The distance from A to B is $10\frac{1}{4}$ blocks. John starts from A and walks $5\frac{1}{2}$ blocks toward B. Then he walks back $2\frac{1}{4}$ blocks.

(*a*) How far is he from B?

(*b*) How far is he from A?

(*c*) How far has he walked?

36. How much change should I receive from a $10 bill if I spend $$6\frac{3}{4}$$?

Find the cost of the following:

37. $$2\frac{1}{2}$$ doz. books at $$$2\frac{1}{2}$$ a dozen.

38. $$3\frac{1}{4}$$ doz. cups at $$$1\frac{3}{4}$$ a dozen.

39. $$\frac{1}{2}$$ doz. caps at $$$1\frac{1}{4}$$ each.

40. $$16\frac{1}{2}$$ qt. of milk at 8 ct. a quart.

41. $$9\frac{1}{4}$$ lb. of meat at 16 ct. a pound.

42. $$3\frac{1}{2}$$ yd. of cloth at $$$1\frac{1}{2}$$ a yard.

43. 10 bu. oats at $$35\frac{1}{2}$$ ct. a bushel.

44. $$6\frac{1}{4}$$ lb. cheese at 18 ct. a pound.

45. $$12\frac{1}{2}$$ qt. of milk at 8 ct. a quart.

QUANTITY	TOTAL COST	COST OF ONE
46. $$2\frac{1}{2}$$ T. of coal	$16.25	?
47. $$5\frac{1}{4}$$ lb. of butter	1.68	?
48. $$4\frac{1}{2}$$ lb. rice	.27	?
49. $$3\frac{1}{2}$$ lb. flour	.14	?
50. $$3\frac{1}{2}$$ lb. sugar	.28	?
51. $$5\frac{1}{2}$$ lb. beef	.88	?
52. $$1\frac{1}{4}$$ lb. lard	.15	?
53. $$2\frac{1}{2}$$ lb. barley	.15	?
54. $$3\frac{1}{2}$$ lb. farina	.21	?
55. $$4\frac{1}{2}$$ lb. prunes	.45	?

56. One store charged 25 ct. a pound for turkey. Another store charged only 21 ct. a pound. How much is saved by buying at the cheaper price a turkey that weighs 12½ lb.?

57. A vessel has completed 25⅞ mi. of a journey of 36¼ mi. How many miles remain?

58. A farmer had 16¼ barrels of apples. He sold all except 4½ barrels. How many barrels did he sell?

59. How many pieces ¼ yd. long may be cut from a piece of ribbon ¾ yd. long?

60. How many packages of pepper each containing ¼ lb. can be filled out of an 8-lb. can?

61. A chest of tea contains 60½ lb. The tea is put into half-pound boxes. How many boxes are needed?

62. How much will 20 lb. of tea cost at 30 ct. a half-pound?

63. Find the cost of ⅔ of a yard of ribbon at $¾ a yard.

64. What is the area of a rectangle ½ ft. long and ⅞ ft. wide? *

65. How many tons of coal at $6½ a ton may be purchased for $39?

66. How many strips of paper 1½ ft. wide are needed to cover a wall 15 ft. long?

* Multiply the length by the width. (See Art. 218.)

67. How many pieces of rope 1¼ ft. long may be cut from a piece 12 ft. long?

68. If a plumber receives $4½ a day, how many days must he work to receive $18?

69. A boy walks at the rate of 1¾ mi. an hour. At that pace how long will it take him to walk 7 miles?

70. How many packages containing 3½ lb. of sugar can be filled from a 70-lb. bag of sugar?

71. If ⅔ of the distance between two villages is 6 mi., what is the entire distance?

72. ¾ of the number of pupils in a school is 90. How many pupils are in the school?

73. If a family uses 1¼ lb. of butter a day, how many pounds will be used in 10 days?

74. Find the cost of ½ doz. pencils at $⅔ a dozen.

75. How many square feet are in a rug 2½ ft. long and 2¼ ft. wide?

76. A rectangular mat is 1¼ ft. long and 2¼ ft. wide. Find its area.

77. ⅚ of Kate's money is 30 ct. How much money has she?

78. A farmer has 12 apple trees. Each tree yielded an average of 2½ bu. of apples. How many bushels of apples did he obtain?

79. A board 11 ft. long is cut into pieces, each piece being 3 ft. long. How many pieces? How much of the board is left?

80. A board 9 ft. long is cut into pieces, each piece being $\frac{1}{3}$ ft. long. How many pieces? How much of the board is left?

81. How many pieces, each $2\frac{1}{4}$ ft. long, can be cut from a board 18 ft. long?

82. How many books costing 1\frac{1}{4}$ each can be bought for $25?

83. If the price of meat is reduced $2\frac{1}{4}$ cents a pound, how much is saved by a family that uses 24 lb.?

84. If a girl saves 1\frac{3}{4}$ a week, how much will she save in a year?   (52 wk. = 1 yr.)

85. A class contains 40 pupils. 25 are girls. The girls form what part of the class?

86. Henry has 80 ct. He spends 20 ct. What part of his money does he spend? What part of his money does he save?

87. 15 min. is what part of an hour?

88. 9 in. is what part of a foot?

89. 4 ft. is what part of 4 yd.?

90. 30 ct. is what part of $1? Of $3?

91. A boy bought a bat for 25 ct. He sold it for 35 ct. What part of the cost of the bat did he gain?

92. Flour is bought at the rate of $5\frac{1}{2}$ ct. a pound and sold at the rate of $6\frac{3}{4}$ ct. a pound. Find the profit on 80 lb.

93. A piece of ribbon contained $7\frac{1}{2}$ yd. $3\frac{1}{2}$ yd. were sold at $\$\frac{1}{2}$ a yard; the remainder was sold at $\$\frac{1}{4}$ a yard. How much was received for the whole piece?

94. Two boys begin to walk from the same place at the same time. One walks $3\frac{1}{2}$ mi. an hour; the other walks $2\frac{7}{8}$ mi. an hour. How much farther will the first boy walk in 4 hr.?

95. A grocer buys 40 lb. of tea at 20 ct. a pound. He sells it at the rate of 18 ct. a half pound. Find his gain.

96. Two packages of coffee, one containing $2\frac{1}{2}$ lb. and the other containing $3\frac{1}{2}$ lb., were sold for 20 ct. a pound. How much was received for them?

97. If 2 oranges cost 5 cents, how much will 20 oranges cost?

98. If 4 oranges cost 10 ct., how many oranges can be bought for 35 ct.?

99. How much is saved by buying 7 lb. of sugar at the rate of $3\frac{1}{2}$ lb. for 18 ct., instead of at the rate of 7 ct. a pound?

100. A boy is trying to make the cover of a box. The cover must be 16 in. wide. The boy has two

boards; one is $12\frac{1}{2}$ in. wide, the other is $5\frac{1}{4}$ in. wide. How much will have to be cut from one of the boards to make the proper width?

134. 1. The cover of a box is made of three pieces of wood. The pieces are $4\frac{5}{8}$ in. wide, $3\frac{1}{4}$ in. wide, and $6\frac{5}{8}$ in. wide, respectively. Find the width of the cover.

2. A carpenter cuts 4 pieces from a strip of molding. The lengths of the pieces are $18\frac{1}{2}$ in., $12\frac{3}{4}$ in., $10\frac{7}{8}$ in., and $9\frac{5}{8}$ in., respectively. Find the total length of the 4 pieces.

3. A strip of wood is $4\frac{1}{2}$ ft. long. From it two pieces, $9\frac{7}{8}$ in. and $9\frac{3}{4}$ in. long, are cut. How long is the strip left?

4. In making a picture frame 4 pieces of molding are used; 2 of them are $16\frac{1}{2}$ in. long each, the other two are $8\frac{1}{2}$ in. long each. What is the total length of the 4 pieces?

5. A bolt of cloth contains $30\frac{1}{2}$ yd. From it two pieces, one containing $12\frac{5}{8}$ yd. and the other containing $15\frac{1}{2}$ yd., are sold at \$4 a yard. How much is received for the cloth sold?

6. How many yards are left? What is its value at \$ $3\frac{1}{2}$ a yard?

7. A number of workmen are laying 400 yd. of track for a trolley road. After they have completed 285¾ yd., how many yards remain to be laid?

8. A wagon containing 2 T. of coal weighs 4785 lb. What part of the total weight is due to the weight of the wagon? (1 T. = 2000 lb.)

9. A printer used the following amounts of paper: 20½ lb., 26¾ lb., 48 lb., 220½ lb. Find the total amount of paper used.

10. What is the cost of 18½ lb. of white paper at 6 ct. a pound?

11. What is the cost of 25½ lb. of white paper at 5¾ ct. a pound?

12. A printer had 600 lb. of ordinary paper in stock. He has used the following quantities: 74½ lb., 28¾ lb., 18½ lb., 60 lb., 27½ lb. How many pounds has he left?

13. A vessel is sailing between two cities which are 341⅛ miles apart. How many miles must the vessel sail after it has completed 247⅝ miles?

14. How many square inches in each of the following rectangular panes of glass?

	LENGTH	WIDTH
(a)	6 in. ×	5½ in.
(b)	6½ in. ×	8 in.
(c)	4½ in. ×	6½ in.
(d)	13½ in. ×	26 in.
(e)	23¼ in. ×	28¼ in.

15. 3 pencils cost 5 cts. Find the cost of 8 dozen pencils.

16. How much is $\frac{7}{8}$ of $450?

17. A factory employs 624 persons. $\frac{5}{8}$ of them are men; the remainder are boys. How many men are employed? How many boys?

18. A trolley company pays a motorman $2.50 a day, and a conductor $\frac{9}{10}$ as much. How much does the conductor receive per day?

19. One boat can sail 143 mi. in 12 hours. Another boat requires $1\frac{1}{9}$ as long a time to sail the same distance. How long does the second boat require?

20. An automobile travels $\frac{3}{5}$ as fast as a train. How far can the automobile travel in the time that the train takes to go 485 miles?

21. A manufacturer paid $1200 a week in wages. He increased the wages of his men 25 ct. on the dollar. How much more money did he have to pay after the increase?

22. If 30 tons of coal cost $127.50, how much will 10 tons cost at that rate? 15 tons? 6 tons?

23. A man receives 58\frac{1}{2}$ for working 18 days. How much does he receive for 6 days' work? For 12 days' work? For 1 day's work?

24. If 36 boxes are required to hold 324 toys, how many boxes are needed for 108 toys of the same size?

25. If 12 yd. of velvet cost $ 27, how much will 3 yd. cost? 8 yd. ?

26. A carpenter receives $21¼ a week. How much will he receive in 7 weeks? In 10 weeks?

27. If a boat sails at the rate of 6¼ mi. an hour how far will it sail in 5 hours? In 10 hours?

28. If a yard of cloth costs $ 1¾, how much will 8 yd. cost? 11 yards?

29. The lumber required to make a desk cost $8¼. How much must be paid for lumber for 16 desks of the same kind?

30. A train travels 12⅛ mi. faster each hour than a steamboat. How much farther than the boat does the train travel in 9 hours?

31. The price of flour is increased from $6 a barrel to $7½ a barrel. How much more must a grocer pay for 120 bbl.?

32. Steak which cost 16 ct. a pound now sells for 22½ ct. a pound. How much more must be paid by a purchaser who buys 90 lb. of steak?

33. The population of a city 30 years ago was 124,500. Today the population is 5½ times as great. What is the population today?

34. A county raised a crop of 250,000 bu. of wheat last year. By improved farming, the county raised 1¾ times as much this year. What was the size of this year's crop?

35. The amount of freight carried by a railroad is now $2\frac{1}{4}$ times as great as before. If the railroad formerly earned $9600 in freight charges, how much does it earn now?

36. The receipts of a store for a week are 174\frac{1}{4}$. Of this amount, $\frac{3}{8}$ is paid for merchandise, $\frac{1}{4}$ for rent, and $\frac{1}{8}$ for clerk hire. The rest is profit. How much is spent for merchandise? For rent? For the clerk? What is the profit?

37. A load of stone weighs 1875 lb. How much does $\frac{4}{5}$ of the load weigh?

38. The total receipts of a baseball game were $486.90. Of this amount, $\frac{5}{6}$ was divided equally among 9 players. How much money did each receive?

39. An agent sells 20 suits of clothes for $30.50 each. He receives $\frac{1}{8}$ of the amount as his commission. How much does he receive?

40. If it costs $$\frac{2}{3}$ an hour to pay for the gas used in a factory, how much will be paid for the gas used in $150\frac{1}{4}$ hrs.?

41. A man works 8 hr. a day for 30 da. How much should he receive if he is paid $$\frac{11}{20}$ an hour?

42. Find the cost of $16\frac{1}{2}$ kegs of nails at $$3\frac{1}{2}$ a keg.

43. A train traveling from A to B requires $10\frac{1}{4}$ hours. If it travels at an average rate of $20\frac{1}{4}$ miles per hour, how far is it from A to B?

44. How much profit is made on 80½ loads of wood, if a profit of $1⅞ is made on each load?

45. The duty on macaroni is 1½ ct. a pound. Find the duty on 1150 lb.

46. Find the cost of 50½ yd. of carpet at $2½ per yard.

47. How much must be paid to ship 850¼ lb. of starch at 1⅛ ct. a lb.?

48. A bushel of wheat weighs 60 lb. What is the weight of 30½ bu.?

49. If an aeroplane flies 32 mi. an hour, how far will it fly in 4⅔ hr.?

50. A clerk, who had been saving $3¾ a week, now saves $5¼ a week. How much more money will he save in 13 wk than formerly?

51. The expenses of a store are reduced from $40¼ a month to $36¾ a month. How much does the storekeeper save in 2 yr. by the reduction?

52. If a fruit dealer buys 240 oranges at the rate of 3 for 5 ct., and sells them at the rate of 2 for 5 ct., how much does he gain or lose?

53. A train was traveling at the rate of 37⅝ mi. an hour. It then increased its speed by 2⅓ mi. an hour. How many miles, at the second rate of speed, did the train travel in 9 hr.?

Find the cost in the following:

54. 250 ft. of lumber at $7\frac{3}{4}$ ct. a foot.

55. 225 lb. of lead at $6\frac{1}{4}$ ct. a pound.

56. 396 lb. of paper at $9\frac{3}{8}$ ct. a pound.

57. 427 books at $$1\frac{3}{4}$ each.

58. 75 doz. chairs at $$30\frac{1}{4}$ a dozen.

59. 174 T. coal at $$6\frac{2}{5}$ a ton.

60. 249 bbl. of flour at $$6\frac{4}{5}$ a barrel.

61. 316 bbl. of potatoes at $$3\frac{1}{4}$ a barrel.

62. 443 bbl. of apples at $$4\frac{3}{4}$ a barrel.

63. 500 bbl. of onions at $$2\frac{1}{4}$ a barrel.

64. 285 lb. of beef at $12\frac{1}{2}$ ct. a pound.

65. 166 boxes of eggs at $76\frac{1}{2}$ ct. a box.

66. 329 melons at $18\frac{1}{2}$ ct. each.

67. $39\frac{1}{2}$ yd. of carpet at $$3\frac{1}{4}$ a yard.

68. $78\frac{1}{4}$ yd. of cloth at $$2.75$ a yard.

69. $50\frac{1}{2}$ yd. of silk at $$2.80$ a yard.

70. $24\frac{3}{4}$ yd. linen at $$1.30$ a yard.

71. $18\frac{3}{4}$ yd. of lace at $$\frac{7}{8}$ a yard.

72. $35\frac{1}{2}$ yd. of linoleum at $$1.20$ a yard.

73. 7250 envelopes Size A at $$2\frac{1}{4}$ per M.*

74. 9500 envelopes Size B at $$2\frac{3}{4}$ per M.

75. 10,750 envelopes Size C at $$3\frac{1}{4}$ per M.

76. 8000 business cards, white, at $$\frac{5}{8}$ per M.

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* $$2\frac{1}{4}$ per M means $$2\frac{1}{4}$ per thousand.

77. 6750 business cards, colored, at $$\frac{3}{4}$ per M.

78. 249 pieces of glass at $28\frac{1}{2}$ ct. each.

79. A profit of $1\frac{1}{2}$ cents is made on each pencil sold. How many pencils must be sold to make a profit of $1.50 ?

80. $85\frac{1}{2}$ lb. of pepper are put up in quarter-pound packages. How many packages are used?

81. $8\frac{1}{2}$ dozen peaches are put into boxes, each box containing $\frac{1}{2}$ dozen. How many boxes are needed?

82. $\frac{3}{4}$ yd. of cord is used for a package. How many similar packages can be tied up with 36 yd. of cord?

83. A furnace requires an average of $1\frac{1}{5}$ tons of coal a week. At that rate 28 tons will last how many weeks?

84. A baker uses an average of $\frac{3}{8}$ lb. of flour for a loaf of bread.

(*a*) How many pounds of flour will be used for 96 dozen loaves?

(*b*) How many loaves can be made from 196 lb. of flour? How much flour will be left?

85. How many baskets, each holding $\frac{2}{3}$ of a bushel, can be filled from a bin containing 150 bushels?

86. How many strips of paper, each $1\frac{1}{2}$ ft. wide, are needed to paper a wall 27 feet long?

87. A tank contains 75 gallons of water. A pipe carries off $2\frac{1}{2}$ gallons a minute. How long does it take to empty the tank?

88. A piece of cardboard is 28 in. wide. How many strips each $3\frac{1}{2}$ in. wide can be cut from it?

89. A piece of cardboard is 30 in. wide. How many strips each $3\frac{1}{2}$ in. wide can be cut from it?

90. A sheet of cardboard is 28 in. wide and 22 in. long. First, it is cut into strips, each strip being $3\frac{1}{2}$ in. wide. How many strips are there? Then each strip is cut into pieces $2\frac{3}{4}$ in. long. How many pieces (small cards) are there?
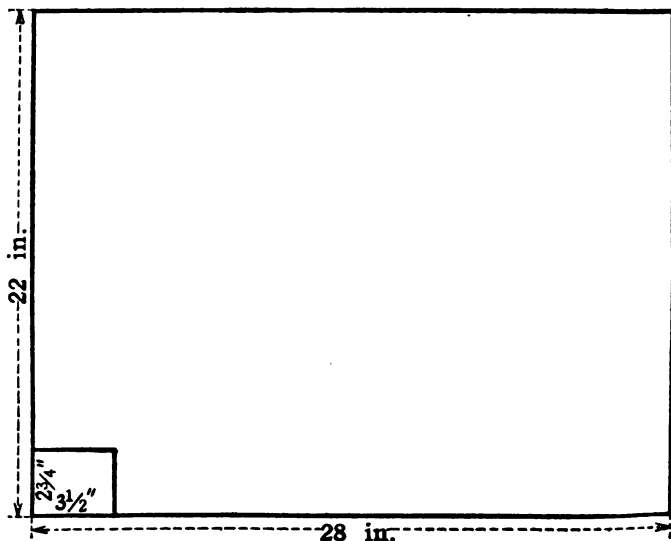
91. How many small cards each $3\frac{1}{2}$ inches long and 2 inches wide can be cut out of a piece of cardboard 28 inches long and 22 inches wide ?

(Make a diagram.)

92. An ordinary brick is 8 in. long and $3\frac{3}{4}$ in. high. How many bricks will be needed to fill a space 32 in. long and 45 in. high ? *

(Make a diagram.)

93. How many tiles, each $2\frac{1}{2}$ in. square, are needed to cover a surface 25 in. long and 30 in. high ?   (Make a diagram.)

94. A printer has an order to supply 500 sheets of business letter paper, each sheet to be $8\frac{1}{2}$ in. long and $5\frac{1}{2}$ in. wide. He cuts the paper from large sheets, each 34 in. long and 22 in. wide.

(*a*) How many sheets of letter paper are obtained from 1 large sheet ?

(*b*) How many large sheets must be used to obtain 500 sheets of letter paper ?

95. The printer has another order to supply 1000 sheets of paper, each $9\frac{1}{2}$ in. long and 6 in. wide. He uses large sheets which are 19 in. long and 12 in. wide.

(*a*) How many small sheets are obtained from each large sheet ?

(*b*) How many large sheets are used to obtain 1000 small sheets?

* Idea of thickness omitted (1 brick thick).

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96. $2\frac{1}{2}$ ft. is what part of a yard?

97. $3\frac{1}{4}$ hr. is what part of a day?

98. $75 is what part of $225?

99. A bushel of wheat weighs 60 lb. 38 lb. are what part of a bushel?

100. 56 lb. is what part of a barrel of flour (196 lb.)?

101. $37\frac{1}{2}$ ct. is what part of $1?

102. $2\frac{1}{2}$ in. is what part of a foot?

103. The floor of a room is 18 ft. wide. What part of the width of the room will be covered by a strip of carpet $1\frac{1}{2}$ ft. wide?

104. John's book contains 282 pages. John has read 48 pages. What part of the book has he read? What part is left for him to read?

105. A train is traveling between two cities which are 240 mi. apart. It has covered 95 mi. of the distance. What part of the distance must it still cover?

106. A toy store had 64 sleds. 20 have been sold. What part of the original number has not been sold?

107. A grocer buys 19 doz. packages of oatmeal. He sells 38 packages. What part of his original stock is unsold?

108. A grocer has 144 lb. of tea. 42 lb. are green tea and the rest is black tea.

(*a*) What part of the total amount is green tea?

(*b*) What part of the total amount is black tea?

109. A plumber receives $26.40 a week. His helper receives only $8.80 a week. The helper's wage is what part of the plumber's wage?

110. A baseball and a glove cost $5. The ball alone costs $1.25.

(*a*) What part of the total expense is for the ball?

(*b*) What part of the total expense is for the glove?

111. Mr. Wilson pays $650 for a horse and wagon. The wagon alone costs $125.

(*a*) State your own question. Find the answer.

(*b*) State another question. Find the answer.

112. A load of coal weighed 2000 lb. after screening. Before screening, the load weighed 2240 lb. What part of the load was waste?

113. What fractional part of a year is the month of June? (365 da. = 1 yr.)

114. The expenses of a store are $150 a month. Of this amount, $3.50 is spent for gas; $37.50 for rent, and the rest for wages.

(*a*) What part of the total expense is for gas?

(*b*) What part of the total expense is for rent?

(*c*) What part of the total expense is for wages?

115. A man receives $200 per month. He saves $137.50 per month.

(*a*) What part of his salary does he save?

(*b*) What part of his salary does he spend?

116. A storekeeper sold $500 worth of shoes. Of this amount, $225 were for men's shoes, $150 for women's shoes, and the rest for children's shoes.

(*a*) What part of the total sales was for men's shoes?

(*b*) What part of the total sales was for women's shoes?

(*c*) What part of the total sales was for children's shoes?

117. If $\frac{1}{8}$ of the population of a city is 1,400,000, what is the population?

118. A factory manufactures 2303 toys in 7 mo. At that rate how many will it manufacture in 12 mo.?

119. If 1 man can do a piece of work in 4 da., in how many days can 6 men do an equal amount of work?

120. If 2 men do a piece of work in $3\frac{1}{2}$ da., how many days will it take 7 men to do the work?

121. If whistles cost $2\frac{7}{10}$ a dozen, how many dozen may be bought for $405?

122. 3 blank books cost 10 ct.

(*a*) How much will 294 books cost?

(*b*) How many books may be bought for $90.50?

123. One dozen bottles of ink cost 50 ct.

(*a*) How much will 356 bottles cost?

(*b*) How many bottles may be bought for $200?

124. A workman receives $ 1½ a day.

(*a*) How much will he receive for working 90 da. ?

(*b*) How many days must he work to receive $ 90?

125. If a boy saves $ ⅝ a week :

(*a*) How much will he save in 52 wk. ?

(*b*) How long will it take him to save $ 96 ?

126. If a storekeeper increases the price of meat 2¼ ct. a pound, how much more money is paid to him for 240 lb. of meat ?

127. The expenses of a store are $ 140 a month. The average daily receipts are $ 3½.   In how many days will the receipts equal the monthly expenses ?

128. The average daily expenses of a store are $ 12¾; the average daily receipts are $ 20½.

(*a*) Find the gain in 30 da.

(*b*) How many days will be required to gain $ 558 ?

129. A pane of glass 8 in. long and 8 in. wide is cut from a larger piece which is 10 in. long and 8 in. wide.

(*a*) What are the dimensions of the piece left over ?

(*b*) What is the area of the piece left over ?
                  (Make a diagram.)

130. A pane of glass 5 in. long and 5 in. wide is cut from a larger piece which is 7 in. long and 5 in. wide.

What is the area of the piece left over?

(Draw a diagram.)

131. A pane of glass $15\frac{1}{2}$ in. long and $10\frac{1}{4}$ in. wide is cut from a larger piece which is 17 in. long and $10\frac{1}{4}$ in. wide.

What is the area of the piece left over?

132. A keg of nails weighs 70 lb. How much will 3500 lb. of nails cost at $\$4\frac{3}{4}$ a keg?

133. Roof shingles are put up in bundles of 250 to each bundle. Find the cost of 9000 shingles at $\$1\frac{1}{4}$ a bundle.

134. A barrel of sugar contained 350 lb. How many packages containing $3\frac{1}{2}$ lb. each can be filled from the sugar in 7 bbl.?

135. If sugar is bought at $\$8\frac{1}{2}$ a barrel and sold at the rate of $3\frac{1}{2}$ lb. for 20 ct., how much money is made on 7 bbl.?

136. A butcher bought 500 lb. of beef at an average price of $10\frac{1}{4}$ ct. a pound. He sold $\frac{1}{4}$ of it at $16\frac{1}{2}$ ct. a pound, $\frac{1}{4}$ at $22\frac{3}{4}$ ct. a pound, and the remainder at 20 ct. a pound. Find his profit.

137. A ton of coal costs $\$6\frac{1}{2}$. A poor family buys coal by the basket, each basket containing 40 lb. How much does the family lose by buying a ton of coal by the basket, if each basket costs 20 ct.? (2000 lb. = 1 T.)

138. If wheat costs $90\frac{2}{3}$ ct. a bushel, how many bushels may be bought for $115.68?

139. If corn costs $34\frac{1}{4}$ ct. a bushel, how many bushels may be bought for $88.36?

140. If oats cost $35\frac{1}{2}$ ct. a bushel, how many · · bushels may be bought for $227.20 ?

141. 27 hammers cost $141\frac{3}{4}$. Find the cost of one.

142. $19\frac{1}{2}$ doz. handkerchiefs cost $14\frac{3}{4}$. Find the cost of 1 doz.

143. $15\frac{1}{2}$ doz. shirts cost $19\frac{3}{8}$. Find the cost of 1 doz.

144. 38 pairs of shoes cost $123\frac{1}{2}$. Find the cost of 1 pair.

145. 18 pairs of shoes cost $76\frac{1}{2}$. Find the cost of 6 pairs.

146. $94\frac{1}{2}$ bu. of wheat weigh 5670 lbs. Find the weight of 1 bu.

147. $70\frac{1}{4}$ doz. caps cost $243\frac{3}{4}$. Find the cost of 1 doz.

148. How many books may be bought for $25.50 at $\$\frac{17}{20}$ each?

149. $10,000 was divided among 3 hospitals. The first hospital received $3500; the second received $2500; the third received the remainder. What part of the total amount was paid to each hospital?

150. The total amount of wheat, corn, and oats produced on a farm was 1800 bu. Of the amount 850 bu. was wheat, and 450 bu. was corn.

(*a*) What part of the total amount was oats?

(*b*) What part of the total amount was wheat?

(*c*) The number of bushels of corn was what part of the number of bushels of wheat?

151. A workman used 150 lb. of sand, 30 lb. of lime, and 15 lb. of hair to make mortar.

(*a*) What part of the total amount was sand?

(*b*) What part of the total amount was lime?

(*c*) What part of the total amount was hair?

152. A number of lamps are packed in a wooden box. The total weight of the lamps and the box is 300 lb. The box weighs $\frac{1}{6}$ as much as the lamps.

(*a*) Find the weight of the box.

(*b*) Find the weight of the lamps.

153. 24 books are packed in a box. The total weight of the books and the box is 77 lb. The box weighs $\frac{1}{20}$ as much as the books.

(*a*) Find the weight of the box.

(*b*) Find the weight of each book.

154. The population of a village is 2340. This is $\frac{1}{12}$ greater than the population was 10 years ago. What was the population 10 years ago?

155. A man sold a horse for $\frac{1}{6}$ more than it cost him. If he sold the horse for $574, how much did he pay for it?

156. George and Harry had a spelling match. George had 90 words correct. This was ⅛ more than Harry had right. How many words did Harry spell correctly?

157. The distance between A and B is 624 miles. This is ¼ more than the distance between A and C. What is the distance between A and C?

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C                         A                         B
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158. A man receives a salary of $3520 a year. This is ⅖ more than he received last year. Find the amount of his salary increase.

159. This year the apple trees in Mr. Jones' orchard yielded 850 bu. This is ⅛ more than they yielded last year. How many bushels did the trees yield last year? What was the total yield in the two years?

160. A grocer has two kinds of coffee. The dearer kind costs ⅖ more than the cheaper kind. If the dearer kind costs 36 ct. a pound, what is the cost of the cheaper kind?

161. An express train travels twice as fast as a local train. If the express train goes 68 mi. an hour, find the speed per hour of the local train.

162. A train travels 4 times as fast as a boat. The train travels 65 mi. an hour. How many miles an hour does the boat travel?

DECIMAL FRACTIONS

135. READING AND WRITING OF DECIMAL FRACTIONS
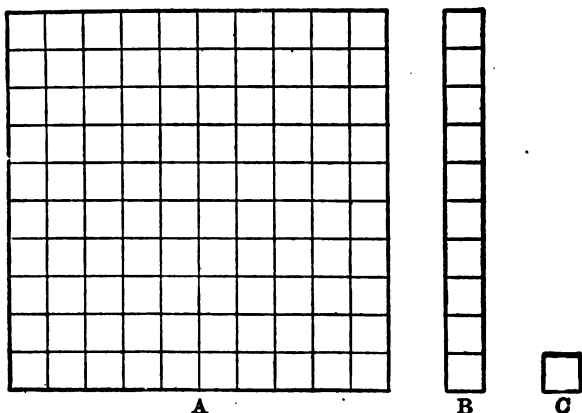
Looking at this picture, answer the following questions:

If C is 1, how much is B? How much is A? Then what would a surface 10 times as great as A be called?

If A is 1, how much is B? How much is C? Then, if C were subdivided into 10 equal parts, what would each part be called?

NOTE TO TEACHER. This chapter is so arranged that operations with decimals of three orders may be treated separately from the work with decimals of more than three orders; or, all orders may be treated together.

Fractions whose denominators are 10, or 100, or 1000, etc., are called **decimal fractions**.

Decimal fractions may be written in the following ways:

(1) in words; *e.g.* three tenths; five hundredths.

(2) as common fractions; *e.g.* $\frac{3}{10}$; $\frac{5}{100}$.

(3) by the use of the decimal point; .3; .05.

Decimal fractions which are written by the use of the decimal point are called **decimals**.

The **decimal point** is a period which separates the orders of whole numbers from the orders of decimal fractions.

The places to the left of the decimal point are called the **orders of whole numbers**: and the places to the right of the decimal point are called the **orders of decimals, or the decimal places**.

The first place to the right of the decimal point is **tenths' place**; *e.g.* .1 is read "1 tenth"; .5 is read "5 tenths."
$$\frac{1}{10} = .1$$
$$\frac{5}{10} = .5$$

The second place to the right of the decimal point is **hundredths' place**; *e.g.* .01 is read "1 hundredth," .25 is read "25 hundredths."
$$\frac{1}{100} = .01$$
$$\frac{25}{100} = .25$$

The third place to the right of the decimal point is **thousandths' place**; *e.g* .001 is read "1 thousandth," .048 is read "48 thousandths," .625 is read "625 thousandths."
$$\frac{1}{1000} = .001$$
$$\frac{48}{1000} = .048$$
$$\frac{625}{1000} = .625$$

The 4th, 5th, and 6th places to the right of the decimal point are respectively **ten-thousandths' place**, **hundred-thousandths' places**, and **millionths' place**. *e.g* .0175 means $\frac{175}{10000}$; .20675 means $\frac{20675}{100000}$; .575283 means $\frac{575283}{1000000}$ or 575,283 millionths.

A **mixed decimal** is a whole number and a decimal written together; *e.g.* 2.75 means $2\frac{75}{100}$; it is read "two and 75 hundredths."

ORDERS OF DECIMAL NOTATION

Nine orders of whole numbers and eight orders of decimal fractions are shown in the following table:

Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths	Hundred-millionths
9	8	7	6	5	4	3	2	1	·	1	2	3	4	5	6	7	8

Orders of Whole Numbers.   Orders of Decimals.

PRINCIPLES. 1. **The value of the orders of decimals** *decreases* from left to right and *increases* from right to left in the same manner as the orders of whole numbers.

2. **The removal of a decimal figure one place to the** *left, multiplies* its value by 10, and its removal one place to the *right, divides* its value by ten; *e.g.* .004 multiplied by 10 becomes .04; and 0.5 divided by 10 becomes .05.

NOTE. Moving the decimal point to the right has the same effect as moving the figure to the left, and moving the decimal point to the left has the same effect as moving the figure to the right.

ORAL EXERCISES

136. Read aloud, giving each decimal the name of its right-hand order; *e.g.* .030 is read " thirty thousandths."

1.	2.	3.	4.	5.
.1	.01	.001	1.1*	20.20
.3	.25	.028	10.5	64.98
.5	.78	.742	406.4	3.003
.9	.50	.030	500.3	300.040
.4	.09	.600	1000.80	60.067

6. In columns 2 to 5, move the decimal points one place to the right and then read the numbers.

WRITTEN EXERCISES

137. Write decimally:

1.	2.	3.	4.	5.
$\frac{4}{10}$	$\frac{26}{100}$	$\frac{75}{1000}$	$\frac{75}{1000}$	$6\frac{30}{100}$
$\frac{9}{10}$	$\frac{45}{100}$	$\frac{1600}{1000}$	$\frac{9}{100}$	$27\frac{5}{1000}$
$\frac{8}{10}$	$\frac{90}{100}$	$\frac{2000}{1000}$	$\frac{425}{1000}$	$600\frac{6}{10}$
$\frac{3}{10}$	$\frac{8}{100}$	$\frac{50}{1000}$	$\frac{71}{100}$	$2\frac{20}{1000}$
$\frac{1}{10}$	$\frac{1}{100}$	$\frac{5}{1000}$	$\frac{50}{100}$	$1005\frac{500}{1000}$

Write decimally the following:

6. Twenty-seven thousandths; thirty-five hundredths; six tenths; seventy-six thousandths.

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* In reading a mixed decimal, the word "and" is used to indicate the place of the decimal point; *e.g.* 2.5 is read "2 *and* 5 tenths"; 285.056 is read "285 *and* 56 thousandths."

7. Fifty hundredths; sixty thousandths; six thousandths; nine-hundred thousandths; nine tenths.

8. 12 and 12 thousandths; 60 and 80 hundredths; 92 and 1 thousandth; 7 and 7 hundredths; 100 and 2 tenths.

9. 307 thousandths; 300 and 7 thousandths; 5 tenths.

10. 800 and 5 hundredths; 200 and 50 thousandths; 10 hundredths.

<div align="center">ORAL EXERCISES</div>

138. Read aloud:

1.	2.	3.	4.
.1234	.12345	.123456	200.0020
.7068	.20640	.260450	4060.00065
.9600	.03090	.100900	900.000900
.0745	.00781	.084010	84.00084
.0109	.42008	.100604	27.0087

5. Read the same numbers, after moving the decimal points *one* place to the right. Two places. Three places.

6. Read the fractions given in the next exercises.

WRITTEN EXERCISES

139. Write decimally:

1.	2.	3.	4.
$\frac{56}{10000}$	$\frac{703}{1000}$	$\frac{2064}{10000}$	$\frac{86}{100000}$
$\frac{450}{100000}$	$\frac{703}{1000000}$	$\frac{2064}{10}$	$\frac{7}{10000}$
$\frac{906}{1000000}$	$\frac{703}{10}$	$\frac{2064}{1000000}$	$\frac{3285}{10000}$
$\frac{2400}{100000}$	$\frac{703}{100000}$	$\frac{2064}{1000}$	$\frac{7060}{100000}$

5. Seventy hundred-thousandths; seven ten-thousandths; seven hundred millionths; seven hundred hundred-thousandths.

6. Eleven thousandths; sixty four hundred-thousandths; four hundred eight thousandths; three thousand ten-thousandths.

7. 76 hundred-thousandths; 7600 thousandths; 207 ten-thousandths; 6020 millionths.

REDUCTION OF DECIMALS

Reduction to Lower Orders

140. 1. Express 5 units as tenths; as hundredths; as thousandths.

2. Write the results as decimal fractions, thus:

$$5. = 5.0 = 5.00 = 5.000$$

3. Express $\frac{3}{10}$ as hundredths and as thousandths; write the results.

4. Express $\frac{75}{100}$ as thousandths and write the result.

5. Compare the following:

COMMON FRACTIONS

$$\frac{5}{10} = \frac{10 \times 5}{10 \times 10} = \frac{50}{100}$$

$$\frac{5}{10} = \frac{100 \times 5}{100 \times 10} = \frac{500}{1000}$$

PRINCIPLE. **Multiplying both terms of a fraction by the same number reduces the fraction to *higher terms* without changing its value.**

DECIMAL FRACTIONS

$$.5 = .50$$
$$.5 = .5000$$

PRINCIPLE. **Annexing ciphers to a decimal fraction multiplies its numerator and denominator by the same number and reduces the fraction to a *lower* order without changing its value.**

Reduction to Higher Orders

ORAL EXERCISES

141. 1. Reduce to tenths: $\frac{40}{100}$; $\frac{50}{100}$; $\frac{60}{3000}$; $\frac{800}{1000}$.

2. Reduce to tenths: .40; .50; .600; .800.

3. How many hundredths in 70 thousandths? In 700 thousandths?

4. Change .500 to hundredths; to tenths.

5. Prove that $.400 = .40\emptyset = .4\emptyset\emptyset$.

PRINCIPLE. **Striking out ciphers from the right of a decimal fraction divides the numerator and denominator by the same number and reduces the fraction to a higher order without changing its value.**

ORAL EXERCISES

142. 1. Reduce to tenths: .60, .800, **14.70**, 20.300.

2. Reduce to hundredths: .160, .500, 25.840, 9.030.

3. Express as thousandths: .5, .56, .12, .06, 5.4, 8.

4. Express as hundredths: .8, .0600, 80.1, 54.07000.

5. Reduce to thousandths: .7450, .16000, .280000.

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6. Reduce to highest decimal form: .06180, .256000.

7. Express as ten-thousandths: .566400, .08, 5.

8. Express as millionths: .9, 4.45, 1.0028, 15.

Decimals reduced to Common Fractions

143. Reduce .25 to a common fraction.

Process. $.25 = \frac{25}{100} = \frac{1}{4}.$

RULE. **To reduce a decimal to a common fraction, write the decimal as a common fraction and reduce to lowest terms.**

WRITTEN EXERCISES

Reduce to common fractions or to mixed numbers in lowest terms:

1.	.75	**2.**	.05	**3.**	.8	**4.**	.35
5.	.60	**6.**	4.50	**7.**	.050	**8.**	.500
9.	.750	**10.**	.24	**11.**	.20	**12.**	3.10
13.	.125	**14.**	.95	**15.**	.875	**16.**	.375
17.	.72	**18.**	.48	**19.**	6.375	**20.**	15.240
21.	8.365	**22.**	20.125	**23.**	.1875	**24.**	.0025
25.	.0375	**26.**	.0040	**27.**	.00625	**28.**	.0325

144. A complex decimal is a decimal containing a common fraction; *e.g.* $.3\frac{1}{3}$ is read 3 and one-third tenths; $.002\frac{2}{5}$ is read 2 and two-fifths thousandths.

Reduce $.3\frac{1}{3}$ to a common fraction.

Process. $.3\frac{1}{3} = \frac{3\frac{1}{3}}{10} = \frac{10}{30} = \frac{1}{3}.$

EXERCISES

Reduce to common fractions or to mixed numbers, in lowest terms:

1.	$.02\frac{1}{2}$	**2.**	$.05\frac{3}{4}$	**3.**	$.33\frac{1}{3}$
4.	$.16\frac{2}{3}$	**5.**	$.042\frac{2}{5}$	**6.**	$1.18\frac{3}{4}$
7.	$.14\frac{2}{7}$	**8.**	$.83\frac{1}{3}$	**9.**	$5.55\frac{5}{9}$
10.	$12.083\frac{1}{3}$	**11.**	$4.44\frac{4}{9}$	**12.**	$.194\frac{4}{9}$

Common Fractions reduced to Decimals

EXERCISE

FOR READING AND DISCUSSION

145. 1. How many tenths are there in 1 ? In 3 ? How many tenths are there in $\frac{1}{5}$ of 3, or $\frac{3}{5}$?

$5)\underline{3.0}$
  $.6$ $\frac{3}{5} = \frac{1}{5}$ of 3, or $\frac{1}{5}$ of 30 tenths, or 6 tenths.

2. How many hundredths are there in 3 ?
How many hundredths are there in $\frac{1}{4}$ of 3, or $\frac{3}{4}$?

$4)\underline{3.00}$ $\frac{3}{4} = \frac{1}{4}$ of 3, or $\frac{1}{4}$ of 300 hundredths, or 75
  $.75$ hundredths.

3. How many thousandths are there in 3 ?
How many thousandths are there in $\frac{1}{8}$ of 3, or $\frac{3}{8}$?

$8)\underline{3.000}$ $\frac{3}{8} = \frac{1}{8}$ of 3, or $\frac{1}{8}$ of 3000 thousandths or
  $.375$ 375 thousandths.

4. Reduce to tenths and explain the process: $\frac{1}{2}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{1}{5}$.

WRITTEN EXERCISES

1. Reduce to hundredths: $\frac{1}{4}$, $\frac{1}{2}$, $\frac{4}{25}$, $\frac{13}{25}$, $\frac{7}{10}$, $\frac{4}{5}$.

2. Reduce to thousandths: $\frac{5}{8}$, $\frac{17}{25}$, $\frac{3}{4}$, $\frac{9}{10}$, $\frac{16}{125}$, $\frac{11}{20}$, $\frac{47}{50}$.

146. 1. Reduce $\frac{27}{125}$ to a decimal.

$$\begin{array}{r} .216 \ Ans. \\ 125\overline{)27.000} \\ \underline{250} \\ 200 \\ \underline{125} \\ 750 \\ \underline{750} \end{array}$$

$\frac{27}{125} = \frac{1}{125}$ of 27.0, or $\frac{1}{125}$ of 27.00, or $\frac{1}{125}$ of 27.000, etc.

27.0 ÷ 125 gives .2 and a remainder.

27.00 ÷ 125 gives .21 and a remainder.

27.00 ÷ 125 gives .216 and no remainder.

2. Reduce $\frac{3}{7}$ to a decimal.

$$\begin{array}{r} 7\overline{)3.000} \\ \overline{.428\frac{4}{7}} \ or \ .429 \end{array}$$

Usually, thousandths is exact enough.

Reduce to decimal form:

3. $\frac{6}{25}$.	4. $\frac{94}{125}$.	5. $\frac{18}{25}$.	6. $\frac{31}{50}$.
7. $\frac{27}{40}$.	8. $\frac{3}{25}$.	9. $\frac{5}{16}$.	10. $\frac{3}{16}$.
11. $\frac{29}{40}$.	12. $\frac{78}{125}$.	13. $\frac{7}{75}$.	14. $5\frac{151}{200}$.
15. $1\frac{67}{125}$.	16. $\frac{21}{80}$.	17. $1\frac{5}{11}$.	18. $2\frac{5}{6}$.
19. $3\frac{8}{9}$.	20. $\frac{123}{175}$.	21. $\frac{71}{250}$.	22. $\frac{77}{80}$.
23. $7\frac{12}{625}$.	24. $\frac{999}{1250}$.	25. $4\frac{15}{16}$.	26. $9\frac{25}{32}$.
27. $10\frac{51}{64}$.	28. $25\frac{5}{32}$.	29. $4\frac{57}{625}$.	30. $2\frac{41}{64}$.

31. Tell how to reduce a common fraction to a decimal fraction. (See Principle, Art. 50.)

Decimal Equivalents of Business Fractions

147. Copy and memorize the following equivalents:

1.	2.	3.	4.
$\frac{1}{2} = .5$	$\frac{1}{4} = .25$	$\frac{1}{3} = .33\frac{1}{3}$	$\frac{1}{8} = .125$
$\frac{1}{5} = .2$	$\frac{3}{4} = .75$	$\frac{2}{3} = .66\frac{2}{3}$	$\frac{3}{8} = .375$
$\frac{2}{5} = .4$	$\frac{1}{20} = .05$	$\frac{1}{6} = .16\frac{2}{3}$	$\frac{5}{8} = .625$
$\frac{3}{5} = .6$	$\frac{1}{25} = .04$	$\frac{1}{12} = .08\frac{1}{3}$	$\frac{7}{8} = .875$
$\frac{4}{5} = .8$	$\frac{1}{50} = .02$	$\frac{1}{16} = .06\frac{1}{4}$	$\frac{1}{40} = .025$

WRITTEN EXERCISES

148. 1. Find the decimal equivalents of $\frac{5}{6}$, $\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{15}$.

2. Reduce to decimal form $\frac{1}{300}$, $\frac{1}{30}$, $\frac{1}{233}$, $\frac{1}{500}$, $\frac{1}{400}$, $\frac{1}{800}$.

Compare the following groups of fractions and their decimal equivalents:

3. $\frac{1}{2}$, $\frac{1}{20}$, $\frac{1}{200}$ **4.** $\frac{1}{3}$, $\frac{1}{30}$, $\frac{1}{300}$ **5.** $\frac{1}{4}$, $\frac{1}{40}$, $\frac{1}{400}$

6. $\frac{2}{3}$, $\frac{2}{30}$, $\frac{2}{300}$ **7.** $\frac{1}{8}$, $\frac{1}{80}$, $\frac{1}{800}$ **8.** $\frac{3}{8}$, $\frac{3}{80}$, $\frac{3}{800}$

Reduce to common fractions, in lowest terms:

9. $.00\frac{1}{2}$ **10.** $.00\frac{1}{4}$ **11.** $.00\frac{1}{3}$ **12.** $.00\frac{2}{3}$ **13.** $.00\frac{3}{8}$

14. Which is greater, $.823$ or $\frac{7}{8}$?

15. Which is less, $\frac{2}{3}$ or $.619$?

16. Which is greater, $\frac{7}{12}$ or $.539$?

ADDITION OF DECIMALS

149. Add 27.12, 216.025, 46.7, 105.87.

Process

```
 27.12
216.025
 46.7
105.87
───────
395.715
```

Explanation. Since only like things can be added, the figures of the same order must be written in the same column. Since ten units in any order equal one unit in the next higher order, we may begin at the right and add as with integers.

RULE. **To add decimals, write like orders under one another and add as with whole numbers.**

Find sums:

1.	2.	3.
258.019	760.028	7248.12
6.026	32.65	6946.005
21.5	200.02	764.887
200.975	7.265	2960.5
6790.54	20.881	738.046

4.	5.	6.
294.863	4000.75	2904.063
468.291	6080.028	486.71
463.876	1111.11	2000.002
590.643	296.884	760.508
875.668	7023.09	2800.28
293.848	476.089	128.3

7.	762.0856	8.	6943.0001	9.	2094.000826
	62 528		72.8094		14.7
	400.04		628.00019		693.00257
	62.0062		7020.7284		3009.418406
	6010.0006		65.040806		26.0019
	632.8008		206.084		7.007

Prove Examples 7 and 8.   (See Page 4, Example 6.)

10.	$179.16	11.	$2000.75	12.	$6947.09
	24.08		11.91		72.08½
	100.50		400.04		700.16
	76.11		7625.94		36.37½
	2019.		72.63		9000.90
	71.40		800.		78.42
	817.06		70.70		100.25

13.	$673.85	14.	$843.76	15.	$176.19
	1000.		90.09		22.09
	943.87		2018.72		2500.
	62.19		46.90		76.14
	4000.		372.62½		186.10
	75.60		40.75		92.75
	4.75		6487.62½		1500.
	116.83		62.50		725.93
	2026.94		9.73		86.08

Prove Example 15.

150. Give answers at sight:

1. Add .6 and .07.
2. How much is .05 plus .3 ?
3. Find the sum of .3, .2, and .08.
4. Add .15 and .008.
5. .09 + .7 = ?
6. 1.6 + 2.05 = ?
7. 25.5 + 6.03 = ?
8. 1.2 + .172 = ?
9. Find the sum of 20.7 and 10.22.
10. Add .3, 1.25, and 2.

151. Change common fractions to decimal equivalents before beginning to add.

Find sums:

1. 26.02 + 311.085 + 425.8 + 7208.033.
2. 695.8 + 289.93 + 323.423 + 793.875.
3. 265.072 + 87.006 + 2019.6 + 872.
4. 338.025 + 92.17 + 200.7 + 927.
5. 2.018 + 75.9 + 263.85 + .82 + 900.27.
6. 266.825 + 27.938 + 864.1 + 900.73 + $1\frac{5}{8}$ + 27.923 + 2009.6.
7. $86\frac{3}{4}$ + 827.93 + 6.834 + .092 + 7962 + $87\frac{1}{2}$ + 837.406.

8. 369.27 + 8.002 + 648.7 + 26.93 + .089
+ 26⅘ + 2916.

9. 1492.7 + 37.186 + 2000.7 + 86.429 + 364.08
+ 272.9 + 863.045.

10. 637½ + 823.19 + 2.964 + 200.02 + 74.009
+ 86,423.

WRITTEN PROBLEMS

152. Estimate answers for the following problems. In Example 1, is 100 T. too large or too small a result? Why? Ask yourself similar questions in relation to the other problems.

1. A foundry furnishes three orders of iron rails, weighing 17.638 T., 16.7 T., and 18.45 T. respectively. What is the total weight?

2. What is the sum of 18 thousandths, 7 tenths, and 62 hundredths?

3. The weight of the water in four tanks is 793.12 lb., 870.005 lb., 649.9 lb., and 704.295 lb., respectively. What is the total weight?

4. An automobile traveled 151.875 mi. on Monday, 145.85 mi. on Tuesday, and 100 mi. on Wednesday. What was the entire distance traveled in the three days?

5. A bank book shows five deposits as follows: $27.50, $50, $17.65, $6, $72.80. Find the amount of the deposits.

WRITTEN EXERCISES

DECIMALS OF MORE THAN THREE ORDERS

153. Find the sum of the following:

1. 263.012 + 27.0075 + 83.7 + 625.003.

2. 8004.02 + 96.006 + 80.00005 + 783.5.

3. 65.0073 + 865.1 + 833 + 6.9462.

4. 3789.005 + 86.993 + 6500 + 27.1 + 364.082
+ 4378.0793 + 86.29438 + 90.75.

5. 732.7586 + 25.70893 + 834.993368
+ 4093.9785 + 86.22 + 8750.

WRITTEN EXERCISES

INVOLVING COMPLEX DECIMALS

154. 1. Add 17.7, $29\frac{3}{5}$, $63.08\frac{1}{3}$, 215.875.

HINT. Reduce complex decimals to thousandths, discarding the fractional remainders.

2. Add .2, $\frac{7}{8}$, .13, $\frac{1}{6}$, .215.

3. Find the sum of 10.14, $21.7\frac{1}{2}$, 63.008, 9.46.

4. 72.018 + 60.5 + $\frac{5}{8}$ + $83\frac{1}{3}$ + 200.14 = ?

5. Find the sum of 82 thousandths, $16\frac{1}{2}$ hundredths, $7\frac{1}{2}$ tenths, and $66\frac{2}{3}$ hundredths.

SUBTRACTION OF DECIMALS

155. Subtract 18.785 from 54.5.

First Process	Second Process
54.500	54.5
18.785	18.785
35.715	35.715

Since only like things can be subtracted, the figures of the same order must be written in the same column. Since ten units in any order equal one unit in the next higher order, we begin at the right and subtract as with integers.

Instead of actually writing ciphers to make both decimals of the same order, as in the First Process, we may imagine the ciphers to be written, as in the Second Process.

ORAL DRILLS

156. **1.** From .75 take .3; .4; .2; .6; .5.

2. From .75 take .18; .25; .35; .12; .19.

3. From 8 take .8; .08; .008.

4. From 200 take .02; .25; .75; .82; .5.

5. From 10 take 7.5; 8.14; 3.75; 1.025.

157. **1.**   **2.**   **3.**
From 172.08   28.325   4026.7
take   36.009   6.9   319.022
        ─────   ──────   ────────

        4.   **5.**   **6.**
From 700.5   294.16   493.01
take   94.17   7.928   388.993
       ──────   ───────   ────────

        7.   **8.**   **9.**
From 2638.6   639.45   2163.317
take 2074.37½   150.738   1847.233⅓
       ────────   ────────   ──────────

Prove Examples 4, 6, and 8.

158. **1.** What is the difference between 16.725 and 9.96 ?

2. What is the difference between 600.06 and 60.917 ?

3. What is the difference between 27.93 and 146.8 ?

4. From 200⅛ take 69.27.

5. From 200 take 2 hundredths.

6. 75 hundredths − 6 thousandths = ?

7. Find the difference between 6 hundredths and 6 tenths.

8. Find the difference between 100 and 1 hundredth.

9. Find the difference between $70\frac{1}{6}$ and 32.08.

10. $190.28 - 76\frac{1}{5} = ?$

WRITTEN PROBLEMS

159. 1. A man's income is \$100 per month. His expenses for three months were \$38.65, \$42.50, \$40$\frac{1}{4}$. What sum remained?

2. A dealer bought a \$450 piano. A reduction of \$62.50 was allowed. What was the actual cost?

3. A locomotive runs at an average speed of 52.355 miles per hour. On an up grade it runs at a speed of 38.48 miles per hour. What is the loss per hour?

4. On the 15th of the month the water in a cistern measured 8.217 feet in depth; on the 25th it measured $6\frac{5}{8}$ feet. What was the amount of the decrease?

5. Find the difference between $8\frac{5}{6}$, and 8 and 903 thousandths.

MULTIPLICATION OF DECIMALS

Multiplication by a Whole Number

160. 1. How much is 6 times 4 ct.? 6 times 4 yd.? $6 \times \frac{4}{5}$?

2. How much is 9 times $\frac{4}{10}$? 9 times .4? $9 \times .04$? $9 \times .004$?

3. Multiply by 7: 12 ct.; $12; 12 ft.; $\frac{12}{10}$; .12; .012; 1.2.

4. 5 tenths multiplied by $5 = ?$

5. 5 hundredths multiplied by $5 = ?$

6. 5 thousandths multiplied by $5 = ?$

7. 13 tenths multiplied by $5 = ?$

8. 13 hundredths multiplied by $5 = ?$

9. 13 thousandths multiplied by $5 = ?$

Multiply .18 by 4.

.18
4
―――
72

Proof. $4 \times \frac{18}{100} = \frac{72}{100}$ or .72.

Multiply 218.55 by 48.

218.55
   48   **Proof.** $48 \times 2\frac{1855}{100} = \frac{1048840}{100}$.

174840   Since hundredths is multiplied by a whole

87420   number, the product is hundredths.

10490.4$\emptyset$

ORAL EXERCISES

161. Find products:

1. $3 \times .25$	2. $4 \times .13$	3. $5 \times .15$
4. $8 \times .27$	5. $2 \times .91$	6. $3 \times .28$
7. $5 \times .18$	8. 3×1.5	9. 2×2.7
10. $4 \times .18$	11. $10 \times .64$	12. 10×2.9
13. $6 \times .005$	14. $8 \times .140$	15. $12 \times .012$
16. $9 \times .008$	17. 4×2.111	18. 7×3.015
19. 4×2.175	20. 10×5.641	21. 11×1.03

WRITTEN EXERCISES

162. Find products:

1. $13 \times .79$	2. $26 \times .84$	3. $47 \times .68$
4. $39 \times .33$	5. $29 \times .86$	6. $49 \times .75$
7. $78 \times .49$	8. $28 \times .51$	9. $41 \times .009$
10. $85 \times .126$	11. $72 \times .937$	12. $58 \times .691$
13. $125 \times .89$	14. $236 \times .128$	15. $350 \times .008$
16. $296 \times .013$	17. $364 \times .79$	18. $209 \times .136$
19. $420 \times .39$	20. $310 \times .495$	21. 48×1.07

22. 35×6.04 **23.** 29×3.14 **24.** 37×6.41

25. 34×2.006 **26.** 28×5.142 **27.** 36×6.902

28. 81×3.129 **29.** 68×1.105 **30.** 93×2.704

31. 87×3.009 **32.** 43×6.129 **33.** 127×1.315

34. 249×5.91 **35.** 493×6.918 **36.** 351×7.198

37. 235×5.008 **38.** 591×7.605 **39.** 396×2.104

40. 984×3.986 **41.** 400×8.102 **42.** 900×8.075

Multiplication by 10, 100, 1,000, etc.

163. Multiply 3.75 by 10 ; by 100 ; by 1,000.

3.75	3.75	3.75
10	100	1000
37.5$\emptyset$	375.$\emptyset\emptyset$	3750.$\emptyset\emptyset$

PRINCIPLE. **The removal of the decimal point *one place* to the *right, multiplies* the number by *10;* moving the decimal point *two places* to the *right, multiplies* the number by 10 times 10, or *100;* and moving the decimal point *three places* to the *right, multiplies* the number by 10 times 100, or *1,000;* and so on. (See Art. 135, Principle 2.)**

Ciphers are annexed whenever necessary to give the product the proper number of decimal places.

ORAL EXERCISES

164. Answer quickly :

Multiply by 10 :   Multiply by 100 :

1. 26.8   **3.** 20.52   **5.** 36.2

2. 3.18   **4.** 7.05   **6.** 315.82

7. 7.5   **8.** 62.846   **9.** 9.325   **10.** 80.007

11. Multiply by 1,000: .427; 5.85; 72.5.

12. Multiply by 1,000: 257.8; 96.04; 281.0564.

WRITTEN EXERCISES

165. Multiply 57.03 by 80.

Short Process. $8 \times 570.3 = 4562.4$.

Find the products by the short process:

1.	40×26.08	**9.**	$2,000 \times 63.8426$
2.	80×7.216	**10.**	$5,000 \times 3.875$
3.	60×16.5	**11.**	$7,000 \times 208.16$
4.	30×315.82	**12.**	$6,000 \times 6.8249$
5.	200×76.009	**13.**	750×46.28
6.	400×72.863	**14.**	$2,200 \times 38.408$
7.	700×6.908	**15.**	$2,500 \times .6258$
8.	500×236.8	**16.**	$4,500 \times .0086$

17. Make up four similar examples.

Multiplication by a Decimal

ORAL EXERCISES

166. How much is 6 times $\frac{1}{10}$? 6 times $\frac{4}{10}$? $8 \times .7$? $7 \times .5$?

How much is 4 times $\frac{1}{100}$? 5 times .03? $7 \times .08$? $9 \times .09$?

What is the product of $\frac{1}{10} \times \frac{1}{10}$? $\frac{3}{10} \times \frac{4}{10}$? $\frac{7}{10} \times \frac{5}{10}$? $.3 \times .9$?

What is the product of tenths by tenths?

What is the product of $\frac{1}{100} \times \frac{1}{10}$? $\frac{75}{100} \times \frac{1}{10}$? $\frac{12}{100} \times \frac{7}{10}$? $.5 \times .05$?

What is the product of hundredths by tenths?

1. Multiply 1.25 by 5.

    Process

$$1.25$$
$$\underline{5}$$
$$6.25$$

       Proof

$$\frac{125}{100} \times 5 = \frac{625}{100} = 6.25.$$

2. Multiply 125 by .05.

    Process

$$125$$
$$\underline{.05}$$
$$6\,25$$

$$125 \times \frac{5}{100} = \frac{625}{100} = 6.25.$$

3. Multiply 1.25 by .5.

    Process

$$1.25$$
$$\underline{.5}$$
$$.625$$

$$\frac{125}{100} \times \frac{5}{10} = \frac{625}{1000} = .625.$$

4. How can we tell how many decimal places to point off in the product?

5. Multiply:

.4	.4	.04	.04	.04	.004	.004
2	.2	2	.2	.02	2	.02

167. PRINCIPLE. The number of decimal places in the product is equal to the sum of the decimal places in the multiplicand and multiplier.

RULE. To multiply one decimal by another, multiply as with whole numbers and point off as many places from the right of the product, as the sum of the decimal places in multiplicand and multiplier.*

NOTE. In pointing off, it is sometimes necessary to prefix ciphers to the product in order to get the proper number of decimal places. See Examples 4, 5, and 6 in the next exercise.

SIGHT EXERCISES

Give products quickly:

1. .5 .3	2. .05 .3	3. .005 .3	4. .12 .3
5. .11 .5	6. .011 .7	7. .08· .8	8. .009 9
9. .05 .6	10. .025 5	11. .012 8	12. .8 .5
13. 1.2 7	14. 12 .5	15. .25 .4	16. 2.5 .4

* TEACHER'S NOTE. The term "pointing off" should be explained, and pupils should have practice in pointing off (a) any given number of places; (b) any denominator, as tenths, thousandths, etc.

168. **1.** Multiply 12.75 by .37.

$$12.75$$
$$\underline{\quad.37}$$
$$8925$$
$$3825$$
$$\overline{4.7175}$$

$1275 \times 37 = 47175$

There are 2 decimal places in the multiplicand and 2 in the multiplier. Therefore $2 + 2$, or 4 decimal places must be pointed off in the product.

Find the products of:

2. 7.5 by .35

3. 1.12 by .28

4. 425 by .87

5. 14.6 by 2.8

6. .227 by 45

7. 127 by 24.3

8. 196 by .007

9. 8.75 by .77

10. 7 by .356

11. 280 by .7

12. 3.47 by 80

13. 3.47 by .08

Multiply:

14. .625 by 1.38

15. .349 by 2.64

16. 72.341 by 25

17. 1800.09 by .075

18. 167.08 by .724

19. 2.0602 by .86

20. 1728 by .0325

21. 23.1 by .08125

22. 27.64 by $8\frac{2}{3}$

23. 8.06 by $20\frac{1}{8}$

24. 72.84 by $66\frac{2}{3}$

25. 927.081 by $10\frac{5}{8}$

ORAL DRILLS

169. 1. Find .2 of 16; of 24; of 1.8; of 2.5; of .015.

2. Find .01 of 25; of 40; of 7.2; of 124; of .003.

3. Find .09 of 16; of 1.2; of .006; of 110; of .21.

4. Find .008 of 12; of 50; of .09; of .9; of .41; of .002; of 1.11.

NOTE. To solve the following, change each decimal fraction to the equivalent common fraction. See Table, Art. 147.

How much is:

5. .66⅔ of 18?	**10.** .80 of 35?
6. .40 of 50?	**11.** .125 of 56?
7. .62½ of 80?	**12.** .75 of 60?
8. .33⅓ of 36?	**13.** .16⅔ of 30?
9. .16⅔ of 42?	**14.** .37½ of 72?

ORAL DRILLS

170. 1. How much is .66⅔ of .01 of 12? How much is .25 of .1 of 36?

2. Find .75 of .001 of 800. Find .625 of .01 of 2400.

3. Find .2 of .03 of 5000. Find .5 of .04 of 7.2.

4. How much is .5 of .375 of 16? How much is .2 of .16⅔ of 60?

DIVISION OF DECIMALS

Division by a Whole Number

ORAL EXERCISES

171. 1. Divide by 2: 8, 8 tenths, .8, 8 hundredths, .08, .008.

2. Divide by 5: .25; .025; .30; .030; 2.5; 4.5.

3. Divide by 2: .5 or .50; .3 or .30; .07 or .070.

4. Divide by 4: .2; .02; .06; .26; 21.; 1.3.

5. Divide by 8: 168 ct.; 120 thousandths; 1400 hundredths.

Find the quotients:

6. $1.68 ÷ 8   **7.** .12 ÷ 8   **8.** 14 ÷ 8

     8)$1.68   8).120   8)14.00

WRITTEN EXERCISES

172. Find quotients:

Annex ciphers to the dividend until the division is exact.

NOTE. Be careful to place the decimal point of the quotient directly under the decimal point of the dividend.

1. 1.5 by 6   **2.** .06 by 8   **3.** 1.3 by 5

4. 2.3 ft. by 4   **5.** $20. by 8

6. 30. by 12   **7.** 166.086 A. by 9

8. 200.256 T. by 8   **9.** 465.871 mi. by 7.

Divide, finding quotients to thousandths:

10. $12.39 by 6   11. 421.5 in. by 12

12. $63.08 by 8   13. $2039 by 8

14. 12.593 by 7   15. 172.35 sq. ft. by 9

16. 275.33 by 11   17. $57.42 by 5

18. 280 oz. by 12   19. $150.32 by 10

Division by 10, 100, 1,000, etc.

173. Using short division, divide .42 by 10; 15.6 by 100; 612.5 by 1000.

10)420	100)15.600	1000)612.5000
.042	.156	.6125
or	or	or
.42 ÷ 10 = .042	15.6 ÷ 100 = .156	612.5 ÷ 1000 = .6125.

Proofs: $10 \times .042 = .42$ $100 \times .156 = 15.6$
$$1000 \times .6125 = 612.5$$

It has already been shown (Art. 163) that to *multiply* a decimal by 10, or 100, or 1000, the decimal point is moved one, or two, or three places to the *right*, respectively. Division is the reverse of multiplication.

RULE. **To *divide* a number by *10*, or *100*, or *1000*, the decimal point is moved *one*, or *two*, or *three places* to the *left*, respectively.**

· Decimal ciphers are prefixed to the quotient whenever necessary to give the proper number of decimal places.

By moving the decimal point to the left, divide :

1. By 10: 21.56; $4.85; .19; .2.

2. By 100: $14.56; 216.4; 1.25; .7.

3. By 100: $2.07; .26; .08; 5.

4. By 10: 1.3; .07; .095; 256.

5. By 1000: $285; $64.35; 253.5; 7.09.

6. By 1000: .7; .085; .005; 72896.

WRITTEN EXERCISES

Division by 20, 30, etc.; 200, 300, etc.

174. Using short division, divide :

1. 18.44 by 20; 2. 456.9 by 300; 3. 79.2 by 4000.

$2\emptyset)\overline{1.8_x44}$ $3\emptyset\emptyset)\overline{4.56_x9}$ $4\emptyset\emptyset\emptyset).\overline{079_x2}$
.922   1.523   .0198

It has already been shown that dividing both
dividend and divisor by the same number does not
change the quotient. (See Art. 35.) In the first
example, we cancel the factor 10; in the second,
the factor 100; and in the third, the factor 1000.
In each divisor the cipher or ciphers are struck
out, while in each dividend the decimal point is
moved to the left as many places as there are
ciphers struck out in the divisor.

175. **WRITTEN EXERCISES**

Find quotients to three decimal places in the following examples.

Divide by 20, 30, 50, 80:

1. $24.58
2. $125.75
3. 208.9
4. 767 lb.
5. $7
6. $31
7. $151.03
8. .9
9. 49 in.

Divide by 400, 700, 1200:

10. $38
11. $725
12. 28.3 T
13. 5
14. $548.50
15. $.90
16. 891
17. 3.5

Divide by 6000, 8000, 12000:

18. 9,071
19. 489
20. $72
21. $8

176. Divide $55.10 by 38.

```
    $1.45
38)55.10
    38
    ──
    171
    152
    ───
    190
    190
    ───
```

Since $\frac{1}{38}$ of a number of cents is to be found, the quotient must be cents.

Divide 523.2 by 36.

$$\begin{array}{r} 14.533\frac{1}{8} \\ 36)\overline{523.200} \\ 36 \\ \hline 163 \\ 144 \\ \hline 192 \\ 180 \\ \hline 120 \\ 108 \\ \hline 120 \\ 108 \\ \hline 12 \end{array}$$

Since $\frac{1}{36}$ of a number of thousandths is found, the quotient must be thousandths.

523.200 ÷ 36 gives 14.533 and a remainder.

Usually, the division is not carried beyond thousandths; therefore the result is written $14.533\frac{12}{36}$, or $14.533\frac{1}{3}$, or more simply 14.533.

WRITTEN EXERCISES

177. Find quotients:

1. 12.744 ÷ 54
2. 152.25 ÷ 2500
3. 172.8 ÷ 72
4. 11.648 ÷ 1600
5. 49.02 ÷ 38
6. 3432.45 ÷ 49
7. 6148.14 ÷ 86
8. 164.983 ÷ 47
9. 968.62 ÷ 38
10. 275.003 ÷ 41
11. 3926.05 ÷ 126
12. 802.005 ÷ 319
13. 467.193 ÷ 1270
14. 298.374 ÷ 2400
15. 196.008 ÷ 3017
16. 156.208 ÷ 1705
17. 471.315 ÷ 1150
18. 200.089 ÷ 1349
19. 641.179 ÷ 4173
20. 836.004 ÷ 8196
21. 496.0209 ÷ 8192
22. 715.295 ÷ 7930

Division by a Decimal

178. Divide:

1. $2)\overline{6}$ 2. $.2)\overline{.6}$ 3. $.02)\overline{.06}$ 4. $.002)\overline{.006}$

5. Explain
$$\begin{cases} .6 \div .2 = \\ .06 \div .02 = \\ .006 \div .002 = \end{cases} \Big\rangle 6. \div 2. = 3.$$

If the divisor is multiplied by 10, what must be done to the dividend to prevent a change in the quotient? If the divisor is multiplied by 100? By 1000? (See Principles, Arts. 50, 52.)

These examples establish the important principle:

Multiplying both dividend and divisor by the same number does not change the quotient.

6. How may   .6 ÷ .2   be changed to 6. ÷ 2.?

7. How may   .06 ÷ .02   be changed to 6. ÷ 2.?

8. How may .006 ÷ .002 be changed to 6. ÷ 2.?

Study carefully the following explanations:

The divisor being a whole number, the division is simple.

9. Divide: 8)17.28
$$\overline{2.16}$$

The divisor being a decimal, the division is not simple.

To make the division simple, the divisor must be changed into a whole number.

The divisor may be changed into a whole number without changing the quotient by multiplying both divisor and dividend by 10, or by moving the decimal point *one* place to the right in both dividend and divisor.

The division is now simple.

10. Divide: .8)17.28

8.)172.8
$$\overline{21.6}$$

The divisor being a decimal, the division is not simple. By changing the divisor to a whole number and moving the decimal point two places to the right in the dividend, the division becomes simple.

11. Divide: $.08)\overline{17.28}$

$8.)\overline{1728.}$
$\underline{216.}$

EXERCISES

1. Change $.8)\overline{.64}$ to $8)\overline{6.4}$ and then divide.

2. Change $.8)\overline{32.}$ to $8)\overline{320.}$ and then divide.

3. Change $.08)\overline{.144}$ to $8)\overline{14.4}$ and then divide.

4. Change $.9)\overline{8.1}$ by multiplying both dividend and divisor by 10, and then divide.

5. Change $.8)\overline{.81}$ by moving the decimal point one place to the right in both dividend and divisor and then divide.

179. RULE.   To divide by a decimal.

1. Change the divisor to a whole number by moving the decimal point to the right of the last figure.

2. Move the decimal point in the dividend as many places to the right as the decimal point was moved in the divisor; if necessary, annex ciphers to the dividend.

3. Divide as with whole numbers.

4. Place the decimal point in the quotient directly above the decimal point in the dividend.

ORAL EXERCISES

Divide:

1. .4)$\overline{.8}$ 2. .3)$\overline{.6}$ 3. .5)$\overline{1.5}$

4. .7)$\overline{3.5}$ 5. .4)$\overline{2.8}$ 6. .7)$\overline{6.3}$

7. .9)$\overline{4.5}$ 8. .6)$\overline{5.4}$ 9. .9)$\overline{10.8}$

10. 1.2)$\overline{14.4}$ 11. 1.1)$\overline{13.2}$ 12. 1.2)$\overline{9.6}$

13. .02)$\overline{.04}$ 14. .02)$\overline{.08}$ 15. .03)$\overline{.12}$

16. .05)$\overline{.55}$ 17. .08)$\overline{.96}$ 18. .07)$\overline{.84}$

19. .09)$\overline{1.08}$ 20. .12)$\overline{1.32}$ 21. .11)$\overline{1.21}$

22. .08)$\overline{.064}$ 23. .5)$\overline{.025}$ 24. .09)$\overline{.081}$

25. .12)$\overline{.108}$ 26. .03)$\overline{.027}$ 27. .12)$\overline{.096}$

28. .05)$\overline{.555}$ 29. .08)$\overline{.168}$ 30. .07)$\overline{.147}$

31. .005)$\overline{.025}$ 32. .007)$\overline{.049}$ 33. .008)$\overline{.072}$

34. .012)$\overline{.0144}$ 35. .012)$\overline{1.440}$ 36. .012)$\overline{.000084}$

Prove Examples 10, 15, 20, 25, and 30.

WRITTEN EXERCISES

180. 1. Divide 2.052 by .36.

$$\begin{array}{r} 5.7 \\ _x36.)\overline{2_x05.2} \\ 180 \\ \hline 252 \\ 252 \end{array}$$

To change the divisor into a whole number the decimal point is moved two places to the right.

To balance the change in the divisor, the decimal point in the dividend is moved two places to the right.

Divide :

2. 2.241 by .27   3. 5.625 by .45

4. 52.394 by .67   5. 70.372 by .73

6. 50.932 by .68   7. 32.572 by .68

8. 33.663 by 6.87   9. 30.156 by 3.59

10. 1.4922 by .829   11. 2.4013 by .649

12. 4.6374 by .786   13. 8.4099 by .867

14. 2.2854 by .39   15. 29.516 by 4.7

16. 53.136 by 7.2   17. .7857 by .873

18. 7648 by .956   19. 2.2555 by 3.47

181. Special Cases: 1. Ciphers annexed in the dividend.
2. Ciphers prefixed in the quotient.

WRITTEN EXERCISES

1. Divide 1.75 by .30125.

$$\begin{array}{r} 56. \\ {}_{x}03125.\overline{)1{}_{x}75000.} \\ 15625 \\ \overline{18750} \\ 18750 \end{array}$$

Explanation. Since the dividend has fewer decimal places than the divisor, it is necessary to add ciphers to the dividend when moving the decimal point to the right.

2. Divide .475 by 7.6.

$$\begin{array}{r} .0625 \\ 7{}_{x}6.{)}_{x}\overline{4.7500} \\ 456 \\ \overline{190} \\ 152 \\ \overline{380} \\ 380 \end{array}$$

Explanation. The quotient must have the same number of decimal places as the dividend: hence, a cipher must be put between the decimal point and the first partial quotient (6). Sometimes two or more ciphers are necessary.

Find quotients:

3. $2.688 \div 568$ 4. $1.7 \div .025$

5. $.48 \div 12.8$ 6. $.0144 \div 7.2$

7. $172.8 \div .012$ 8. $3.75 \div .0025$

9. $21.6 \div .0002$ 10. $5.2 \div .013$

EXERCISES

182. Work mentally, using pencils only when numbers are too large to be carried in the mind.

Divide:

1. .21 by 3; by 7; by 21; by 2; by 4.

2. .42 by 2; by 4; by 6; by 7; by 12.

3. .375 by 3; by 5.   6. 3.65 by 2; by 5.

4. .750 by 3; by 5.   7. 18.06 by 2; by 3.

5. .84 by 7; by 3.   8. 6.008 by 4; by 8.

Find quotients:

9. $9)\overline{28.89}$ 14. $7)\overline{49.063}$ 19. $10)\overline{73.681}$

10. $7)\overline{37.24}$ 15. $9)\overline{38.88}$ 20. $20)\overline{39.416}$

11. $4)\overline{35.72}$ 16. $3)\overline{123.126}$ 21. $100)\overline{81.749}$

12. $8)\overline{40.016}$ 17. $5)\overline{64.005}$ 22. $200)\overline{95.120}$

13. $5)\overline{31.005}$ 18. $8)\overline{33.240}$ 23. $500)\overline{58.300}$

Divide:

24. .5)$\overline{.15}$ **30.** 1.2)$\overline{2.4}$ **36.** 10)$\overline{7.34}$

25. .4)$\overline{.12}$ **31.** 3.5)$\overline{7.}$ **37.** .1)$\overline{7.34}$

26. .7)$\overline{.28}$ **32.** .09)$\overline{.36}$ **38.** 100)$\overline{69.3}$

27. .9)$\overline{.036}$ **33.** .04)$\overline{132}$ **39.** .01)$\overline{69.3}$

28. .8)$\overline{.104}$ **34.** .16)$\overline{4.8}$ **40.** 1000)$\overline{4.92}$

29. .6)$\overline{1.8}$ **35.** 25.)$\overline{3.75}$ **41.** .001)$\overline{4.92}$

Simplify, using cancelation:

42. $\dfrac{.3}{.6} = ?$ **48.** $\dfrac{.06}{.03} = ?$ **54.** $\dfrac{.2 \times .4}{.4} =$

43. $\dfrac{.9}{.3} = ?$ **49.** $\dfrac{.06}{.3} = ?$ **55.** $\dfrac{.3 \times .9}{.6} =$

44. $\dfrac{.15}{.5} = ?$ **50.** $\dfrac{.06}{3} = ?$ **56.** $\dfrac{.02 \times .04}{.4} = ?$

45. $\dfrac{.15}{.05} = ?$ **51.** $\dfrac{.25}{.05} = ?$ **57.** $\dfrac{3 \times .6}{.3} = ?$

46. $\dfrac{.15}{5} = ?$ **52.** $\dfrac{2.5}{.05} = ?$ **58.** $\dfrac{3 \times .06}{.3} = ?$

47. $\dfrac{.15}{50} = ?$ **53.** $\dfrac{25}{.05} = ?$ **59.** $\dfrac{.03 \times .06}{.3} = ?$

WRITTEN EXERCISES

183. Divide:

1. .135 by 9; by 5.

2. .232 by 4; by 8.

3. .318 by 3; by 6.

4. .376 by 4; by 8.

5. .418 by 3: by 9.

6. 1.008 by 4; by 8.

7. 4.576 by 4; by 16.

8. 8.04 by 5; by 20.

9. 14.006 by 5; by 1.5.

10. 29.314 by 12: by 1.6.

Find quotients:

11. $11.28 \div 38$

12. $35.09 \div 61$

13. $7.003 \div 29$

14. $.126 \div 3.5$

15. $1.205 \div .64$

16. $1.902 \div 1.18$

17. $506.1 \div 22.9$

18. $4.131 \div 30\frac{3}{4}$

19. $26.47 \div 301$

20. $91.20 \div .426$

21. $36.29 \div .007$

22. $8.416 \div 1.001$

23. $.231 \div .00\frac{1}{4}$

24. $2.84 \div .06\frac{1}{2}$

25. $9.54 \div .0\frac{1}{8}$

26. $1.513 \div .0602$

27. $3120 \div 3.25$

28. $3120 \div .325$

29. $3120 \div .03\frac{1}{4}$

30. $3120 \div 32.5$

31. $461.1 \div 1.74$

32. $379.44 \div 30.6$

33. $33.235 \div .289$

34. $207.483 \div 3.007$

35. $71.447 \div 19.31$

36. $.023481 \div 2.609$

37. $.220626 \div 3.1518$

38. $.360828 \div 9.0207$

39. $1.5084 \div 1.0056$

40. $12.2514 \div 7.0008$

o

Perform the operations indicated by the signs:

41. $17 \times 1.8 \times .006$

42. $\dfrac{35.21}{1.16}$

43. $14.09 + 3.6 + 21.5 - .008$

44. $\dfrac{.18 \times .06 \times .05}{.15 \times .03 \times .09}$

45. $(4.5 + 9.7) + 2.5$

46. $6.503 + (7.2 \times 3.8)$

47. $(3 \times .009) + (6 \times .009)$

48. $(9 \times 3.146) - (7 \times 3.146)$

49. $(15 \times 2.008) - (8 \times 2.008)$

50. $(18 \times .0103) + (6 \times .0103)$

51. $.003\tfrac{1}{4} + 3\tfrac{1}{4}$

52. $2.005 + 8.06 - 7.009$

TYPE PROBLEMS IN DECIMAL FRACTIONS

In Art. 127, we learned that there are three types of problems involving multiplication and division of fractions. The same three types of problems are found in multiplication and division of decimal fractions.

First Type Problem.   To Find a Part of a Number

184. COMMON FRACTIONS.   A carpenter receives $24 for a week's work.   How much does he receive for working $\frac{5}{6}$ of a week?

DECIMAL FRACTIONS.   A laborer mixed 250 lb. of mortar; .84 of it was sand.   How many pounds of sand were used?

Explanations

COMMON FRACTIONS	DECIMAL FRACTIONS
The amount received for $\frac{5}{6}$ of a week is $\frac{5}{6}$ of the amount received for a week. Therefore the amount is	The amount of sand is .84 of the amount of mortar. Therefore the amount of sand is .84 of 250 lb. or
$\frac{5}{6}$ of $24 = $20 *Ans*	.84 × 250 lb. = 210 lb. *Ans*

The first problem is solved by multiplication of common fractions; the second by multiplication of decimals.

HINT. Solve the problem by the use of common fractions whenever the decimal fraction can be easily reduced to a common fraction.

Second Type Problem. To Find What Fraction One Number is of Another Number

185. COMMON FRACTIONS. A storekeeper had 70 packages of oatmeal; he sold 35 packages. What part of his stock did he sell?

DECIMAL FRACTIONS. A storekeeper had 74 packages of oatmeal; he sold 35 packages. What decimal part of his stock did he sell?

• Explanations

COMMON FRACTIONS	DECIMAL FRACTIONS
The part sold is 35 packages out of 70 packages or $\frac{35}{70}$ of the total stock.	The part sold is 35 packages out of 74 packages, or $\frac{35}{74}$ of the total stock.
Therefore the answer is $\frac{35}{70} = \frac{1}{2}$	Therefore the answer is $\frac{35}{74} = .47\frac{11}{37}$

Third Type Problem. To Find a Number when a Fractional Part is Given

186. COMMON FRACTIONS. $\frac{3}{5}$ of the number of pupils in a class is 27. How many pupils are there in the class?

DECIMAL FRACTIONS. .76 of the population of a town is 4066. What is the population?

Explanations

COMMON FRACTIONS	DECIMAL FRACTIONS
$\frac{3}{5}$ of the number of pupils is 27. Therefore, $\frac{1}{5}$ of the number of pupils equals $\frac{1}{3}$ of 27.	.76 of the population equals 4066. Therefore, .01 of the population equals $4066 \div 76$ (or $\frac{1}{76}$ of 4066).
The whole number of pupils equals 5 times ($\frac{1}{3}$ of 27) or $\frac{5}{3}$ of 27 = 45 *Ans.*	The total population equals 100 times $(4066 \div 76)$ or 5350 *Ans.*
A shorter explanation is: $\frac{3}{5}$ of the number of pupils equals 27.	A shorter explanation is: .76 of the population equals 4066. Therefore the total population equals
The whole number of pupils equals $27 \div \frac{3}{5}$ or $27 \times \frac{5}{3}$ = 45 *Ans.*	$4066 \div .76$ or 5350 *Ans.*

WRITTEN PROBLEMS

187. 1. A book which cost $1.50 was sold at a gain of .35 of its cost.

(*a*) Find the gain.

(*b*) Find the selling price.

2. A wagon which cost $ 125 was sold at a loss of .24 of the cost.

(*a*) Find the loss.

(*b*) Find the selling price.

3. 780 tons of coal were shipped to New York. .68 of it was hard coal; the rest was soft coal. How many tons of each kind were there ?

4. A manufacturer bought 86 tons of coal. He used .32 of it.

(*a*) How many tons were used ?

(*b*) How many tons were not used ?

5. The distance from New York to Buffalo by way of Albany is 439.7 mi.   The distance between New York and Albany is .33 of that distance.

(*a*) How far is it from New York to Albany?

(*b*) How far is it from Albany to Buffalo?

6. 5 in. is what decimal part of 3 ft. ?

7. 1350 lb. is what decimal part of a ton ?

8. A grocer sold 226 lb. of tea.   100 lb. of it was black tea.   What part of the total amount (expressed in decimals) was black tea ?

9. A fur coat marked at $465 was reduced to $375.   What reduction (expressed decimally) was made in the price of the coat?

10. An automobile party are making a trip of 950 mi.   They have gone 875 mi.   What part of the entire distance (expressed decimally) has been covered ?

11. .54 of the height of the Metropolitan Building, New York, is 426.6 ft.   How high is the building?

12. From 1900 to 1910 the population of a city increased .22.   The actual increase was 115,550 persons.   Find the population in 1900 and the population in 1910.

13. .18 of the amount of money received for an entertainment is $657.90.   What was the total amount received ?

14. A number of barrels of sugar were sent to a wholesale dealer. .46 of the total amount was granulated sugar. The rest was loaf sugar. There were 851 bbl. of granulated sugar.

(*a*) How many barrels were shipped in all?

(*b*) How many barrels were loaf sugar?

15. The wages of the employees in a factory were increased .12. The manufacturer paid $318.03 more than formerly on account of the increase. Find the total amount formerly paid for wages.

16. The population of Chicago is approximately 2,185,000; the population of Boston is approximately 670,000. The population of Boston is what decimal fraction of the population of Chicago?

17. Harry weighs 150 lb. William weighs .72 as much as Harry. How much does William weigh?

Estimate William's weight before trying to solve the problem. Is your estimate more or less than 75 lb.? Why?

18. One river is 1400 mi. long. Another is 1.4 as long. How long is the second river?

Estimate the answer. Is it more or less than 1400 mi.? Why?

19. A horse is sold at a gain of .36½ of its cost. The gain is $73. Find the cost.

20. A number of barrels of beef weighed 1632 lb.; .02 of the weight was due to the wooden barrels. Find the actual weight of the beef.

21. In making coins, a government used 5 parts pure silver and 2 parts alloy. What part of the coin (expressed decimally) was pure silver?

22. 7½ lb. is what part of 196 lb. (expressed decimally)?

23. If a train travels .15 of its journey in 13.5 hr., how many hours are required for the entire journey?

24. If a train travels .08 of its journey in 1½ hr., how many hours will be required for the rest of the journey?

25. A railroad carried 76,500 passengers in October. The number carried in November was .16 greater than in October. The number carried in December was .15 greater than in November.

(*a*) How many passengers were carried in November?

(*b*) How many passengers were carried in December?

(*c*) How many passengers were carried in the 3 months?

26. A factory manufactured 1500 toys during the first year, .32 more during the second year than during the first year, and .35 less during the third year than during the second year.

(*a*) How many toys were manufactured during the second year?

(*b*) How many toys were manufactured during the third year?

27. It cost $25,600 to build a house. .36 of the cost was paid for materials; the remainder, for labor. How much was spent for materials? How much for labor?

28. .48 of the cost of a house is $17,405.60. Find the cost.

29. .36 of a farmer's crop of wheat was spoiled by frost. The remainder amounted to 5568 bu. How many bushels were destroyed?

30. One candidate for an office polled 7500 votes; another polled 1.8 times as many votes. How many votes were cast for the second candidate?

31. A meter is 39.37 in. How many inches in 150 meters?

32. A man measures the length of a plot of ground in rods. He finds that there are 9.5 rods. Give the length of the plot in feet.

33. How many fence boards 9.3 ft. long will be required in fencing a rectangular plot of ground 49.5 ft. long and 40.25 ft. wide?

34. A working girl spends $.15 a day. In how many days will she spend $2.25?

Find the cost in each of the following:

   35. 26 pencil boxes at $.24 each.

   36. 93 lamp shades at $.85 each.

   37. 36 magazines at $.15 each.

   38. 72 blank books at $.39 per dozen.

   39. 45 paper boxes at $.26 each.

   40. 17 pictures at $.88 each.

   41. 39 rulers at $.14 each.

   42. 27 brushes at $.73 each.

   43. 35 brooms at $.81 each.

   44. 68 combs at $.37 each.

   45. 24 chimneys at $1.14 per dozen.

   46. 41 mirrors at $1.92 each.

	AMOUNT PAID	COST OF ONE	NUMBER PURCHASED
47.	$18.90	$1.05	?
48.	85.32	3.16	?
49.	95.45	.83	?
50.	21.42	.17	?
51.	20.40	.24	?
52.	143.96	2.36	?
53.	223.60	5.20	?
54.	31.50	.42	?
55.	61.62	.79	?
56.	38.15	1.09	?
57.	51.48	.99	?
58.	20.64	1.29	?

PROBLEMS IN QUANTITY AND COST

BUSINESS FRACTIONS

188. Review Art. 147.

Express in cents the following fractional parts of a dollar: $\frac{1}{8}$, $\frac{2}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{6}{8}$, $\frac{7}{8}$, $\frac{9}{8}$.

How many cents in $\$\frac{1}{3}$? $\$\frac{2}{3}$? $\$\frac{1}{6}$? $\$\frac{5}{6}$? $\$\frac{4}{3}$? $\$\frac{7}{6}$?

Reduce to cents: $\$\frac{1}{5}$, $\$\frac{2}{5}$, $\$\frac{3}{5}$, $\$\frac{4}{5}$, $\$\frac{1}{10}$, $\$\frac{3}{10}$, $\$\frac{7}{10}$, $\$\frac{9}{10}$.

How many cents in $\$\frac{1}{20}$? $\$\frac{1}{16}$? $\$\frac{1}{12}$? $\$\frac{1}{9}$?

TABLE OF EQUIVALENTS

Copy and memorize the following:

50 ct. = $\$\frac{1}{2}$	20 ct. = $\$\frac{1}{5}$	30 ct. = $\$\frac{3}{10}$	62$\frac{1}{2}$ ct. = $\$\frac{5}{8}$
25 ct. = $\frac{1}{4}$	40 ct. = $\frac{2}{5}$	70 ct. = $\frac{7}{10}$	87$\frac{1}{2}$ ct. = $\frac{7}{8}$
33$\frac{1}{3}$ ct. = $\frac{1}{3}$	60 ct. = $\frac{3}{5}$	90 ct. = $\frac{9}{10}$	16$\frac{2}{3}$ ct. = $\frac{1}{6}$
66$\frac{2}{3}$ ct. = $\frac{2}{3}$	80 ct. = $\frac{4}{5}$	12$\frac{1}{2}$ ct. = $\frac{1}{8}$	5 ct. = $\frac{1}{20}$
75 ct. = $\frac{3}{4}$	10 ct. = $\frac{1}{10}$	37$\frac{1}{2}$ ct. = $\frac{3}{8}$	4 ct. = $\frac{1}{25}$

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189. 1. How much will 24 readers cost at 62$\frac{1}{2}$ ct. each?

Solution. At $\$1$ each, 24 readers would cost $\$24$.

Therefore, at 62$\frac{1}{2}$ ct. each, 24 readers will cost $\frac{5}{8}$ of $\$24$, or $\$15$.

Without using pencil or pen find the cost of:

1. 32 doz. at 87½ ct. a dozen.

2. 200 clocks at 75 ct. each.

3. 150 lb. at 20 ct. a pound.

4. 80 gr. buttons at 37½ ct. a gross.

190. Find the cost of 25 bu. of turnips at 36 ct. a bushel.

Solution. 100 bu. would cost $36.
Therefore, 25 bu. would cost ¼ of $36 or $9.

ORAL PROBLEMS

Find tne cost of:

> **1.** 75 T. at $6 a ton.
>
> **2.** 87½ bu. at 72 ct. a bushel.
>
> **3.** 125 collars at 24 ct. each.
>
> **4.** 25 boxes at 42 ct. each.
>
> **5.** 33⅓ lb. at 24 ct. a pound.
>
> **6.** 37½ doz. at 48 ct. a dozen.

ORAL PROBLEMS

191. **1.** At 12½ ct. each, how many oranges can be bought for $5?

Solution. 12½ ct. is ⅛ of a dollar; therefore $1 will buy 8 oranges, and $5 will buy (5 × 8) or 40 oranges.

2. At 25 ct. a yard, how many yards of oilcloth can be bought for $8?

$12 will pay for how many

3. Pictures @ 33⅓ ct.?   **4.** Books @ 50 ct.?

5. Handkerchiefs @ 12½ ct.? **6.** Chimneys @ 10 ct.?

7. Collars @ 20 ct.?   **8.** Pencils @ 5 ct.?

WRITTEN PROBLEMS

192. 1. At $1.87½ per yard, how many yards of silk can be bought for $250?

$$\text{Process.} \quad \overset{50}{\cancel{250}} \times \frac{8}{\underset{3}{\cancel{15}}} = \frac{400}{3} = 133\tfrac{1}{3} \text{ yd. } Ans.$$

2. How many chairs at $8⅓ can be bought for $50?

3. At $1.25 each, how many books can be bought for $35?

4. How many weeks' board can be paid for with $150, if board costs $12½ a week?

5. How many packages of sugar each weighing 7½ lb. can be made up from 300 lb.?

193. Find the cost of carpet as follows:

60 yd. @ $1.25.		80 yd. @ $1.33⅓.	
Process	Explanation	Process	Explanation
$60	60 yd. @ $1	$ 80	80 yd. @ $1
+15	60 yd. @ $¼	+26⅔ ...	80 yd. @ $⅓
$75	60 yd. @ $1.25	$106⅔ ...	80 yd. @ $1.33⅓

Find the cost of 45 qt. of olive oil

@ 75 ct.		@ 83⅓ ct.	
Process	**Explanation**	**Process**	**Explanation**
$45	.... 45 qt. @ $1	$45	.... 45 qt. @ $1
−11¼	.... 45 qt. @ $¼	−7½	.... 45 qt. @ $⅙
$33¾	.... 45 qt. @ $¾	$37½	.... 45 qt. @ $⅚

WRITTEN PROBLEMS

Find the cost of:

1. 72 books @ 87½ ct.

2. 45 combs @ $1.12½.

3. 36 fountain pens @ $2.37½.

4. 60 chairs @ $5.75.

5. 24 desks @ $12.75.

6. 30 hats @ $4.87½.

7. 42 clocks @ $1.66⅔.

8. 50 chimneys @ 37½ ct.

9. 2 doz. table covers @ $3.62½ per doz.

10. 30 pr. gloves @ 83⅓ ct.

11. Make up five problems similar to those given above.

BILLS

194. The following is a common form of bill:

CHARLES FRISSELL
City Market
268 WEST 42D STREET

New York,____Dec. 27,____1910

Sold to____Mrs. Richard Lewis,_____

_____100 West 45th St., City._____

Dec.	20	2⅞ lb. Steak	24¢		69		
		4 lb. Butter	38¢	1	52		
		1 doz. Oranges	30¢		30		
		3 doz. Eggs	60¢	1	80		
	22	14¾ lb. Turkey	28¢	4	13		
		3 qt. Cranberries	15¢		45		
		1 b. Celery	18¢		18		
		3 lb. Nuts	19¢		57		
		½ doz. Lemons	25¢		13	9	77

Received payment,

Chas. Frissell.

Per A. Bruce.

A bill should show:

1. The place and date of sale.

2. The name and address of the buyer, or **debtor**.

3. The name and address of the seller, or **creditor**.

4. The quantity, name, and price of each item, and the entire amount for each item.

5. The total amount or **footing**.

The creditor sends the bill to the debtor. When the debtor pays the bill, he usually gets from the creditor a receipt or acknowledgment that the bill has been paid. In most cases acknowledgment is made on the bill itself by the creditor or his representative, who signs his name after the words " Received payment " or " Paid." If a representative of the creditor receives the money, he usually signs his employer's name and then his own name preceded by the word " per " or "by."

WRITTEN EXERCISES

Note. The teacher should furnish the pupils with printed or mimeographed bill forms for the following work.

Fill in the blanks with the following items:

1. Mrs. Charles Lane bought of Park and Tilford, 60th Street and 5th Avenue, New York, the following bill of goods:

10 lb. Star Brand coffee at 30 ct. per lb.; 15 lb. loaf sugar at 6 ct.; 3 lb. rice for 25 ct.; ½ doz. cans

White Brand corn at $1.00 per dozen. The bill was paid on delivery.

2. Mr. Smith, the athletic director of your school, purchased the following goods from A. G. Spaulding & Bro., 118 Nassau St., New York:

4 baseballs at $4.50 per dozen; 3 bats at $4 per doz.; 1 basketball for $4.75; 1½ doz. athletic shirts at $2.25 per dozen. The bill was paid one month after the goods were delivered.

3. The Board of Education bought on account of your school, the following supplies from the School Supply Company, St. Louis, Mo.:

50 gro. pen points at 27 ct. per gross; 100 doz. pads yellow ruled paper at 22 ct. per dozen; 15 blackboard compasses at $3 per dozen; 20 1-lb. cans of ink crystals at $6 per dozen. Terms, 3 months.

4. Your principal ordered for your class the following books from the Macmillan Company:

48 Van Wagenen's Dictation at 18 ct.

50 Channing's History at 71 ct.

46 Baker-Carpenter's Fifth Reader at 48 ct.

40 Tarr-McMurry's First Book in Geography at 65 ct.

Make out the bill that your principal received for these books.

P

5. The Wanamaker Company, Philadelphia, Pa., sold to Henry Thorpe, 142 Chestnut Street, Philadelphia:

> 35 yd. Brussels Carpet   at $1.65
> 15 yd. Axminster Carpet at 3.25
> 3 Rugs 3′ × 4′   at 2.85
> 125 yd. Filling   at   .64½

Make out the bill.

6. Suppose that your grocer bought the following merchandise from A. B. Conant & Co., Chicago, Ill., and make out the bill:

5 bbl. sugar at $2\frac{3}{4}$ ct. per lb., total weight 1250 lb.

125 boxes starch at $1\frac{3}{4}$ ct. per lb., total weight 5180 lb.

2 cases indigo at 42 ct. per lb., total weight 105 lb.

150 bags coffee, 50 lbs. each, at $21\frac{1}{8}$ ct. per lb.

Terms 60 da.

7. Thomas Hamilton, Plumber, repairs the furnace of Andrew T. Jackson and sends a bill for the following supplies and services:

1 new firepot, $14; 1 water pan, $3.25; 4 17-in. stove rods, 16 ct.; 6 $1\frac{1}{4}$-in. bolts, 20 ct.; 2 cans furnace cement for 60 ct.; 2 pails mortar for 50 ct.; ½ pail plaster for 20 ct.; 1 can black varnish, 25 ct.; service of plumber, $1\frac{1}{4}$ da. at $3.50 per day; helper, $1\frac{1}{4}$ da. at $1.50 per day. Make out the bill.

WRITTEN EXERCISES

Make and receipt bills for the following sales to yourself:

1. From Park and Tilford, Grocers, for

5 lb. Coffee @ 22 ct., ½ lb. Tea @ 75 ct.,
6 doz. Eggs @ 58 ct., 2 doz. Lemons @ 18 ct.,
6 jars Jam @ 38 ct., 10 lb. Sugar @ 6½ ct.

2. From Hegeman &. Co., Druggists, for

1 bottle Cod Liver Oil, 60 ct.,
1 Toothbrush, 25 ct.,
2 lb. Absorbent Cotton @ 35 ct.,
3 doz. Quinine Pills @ 15 ct.

3. From Mary Anderson, Dressmaker, for

Making of Dress, $18, Trimmings, $7.48,
Findings, $3.82, Lace, $4.75.

Make out bills as follows:

4. From a doctor for professional services.

5. From a plumber.

6. From a carpenter.

7. From a harness-maker.

8. From a dealer in vegetables.

9. From a milliner.

10. From a typist.

DENOMINATE NUMBERS

A **denominate number** is a number that is composed of defined units of measure; *e.g.* 3 feet, 5 quarts.

Long Measure

195. Long measure, or linear measure, is used in measuring lengths or distances. The standard unit of length is the yard. The standard yard is a metal bar carefully preserved by the government in Washington. An exact copy is kept in each state capital.

Table of Long Measure	
12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
5½ yards	= 1 rod (rd.)
16½ feet	= 1 rod
320 rods	= 1 mile (mi.)
1760 yards	= 1 mile
5280 feet	= 1 mile

One eighth of a mile is sometimes called a furlong.

Circular Measure

196. Circular measure is used in measuring angles.

If two lines start from the same point and take different directions, they form an **angle**; *e.g.* the lines *A* and *B* start from the point *P* and take different directions. The point *P* is called the **vertex** of the angle; and the lines *A* and *B* are called the **sides** of the angle.
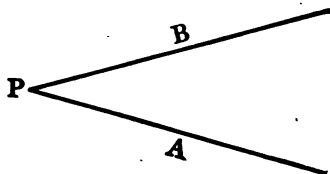
The size of the angle depends on the amount of difference of direction of the sides.

The size of the angle does not depend on the lengths of the sides.

In order to measure the size of angles we consider them to be formed by the straight line *C1* rotating about the point *C* without leaving the surface of the page. Then we measure the difference of direction of each new position of the rotating line with its first position. Thus *C2* forms
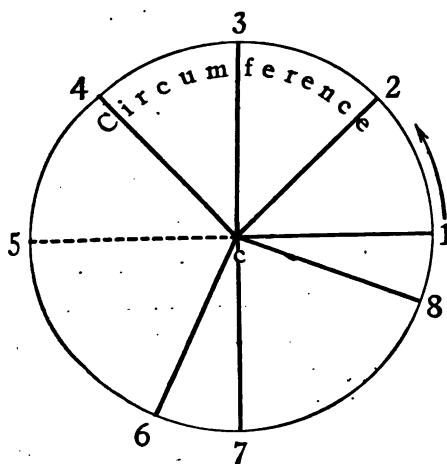
a small angle with *C1*. Larger and larger angles are formed with *C1* as the rotating line takes in turn positions *C3*, *C4*, *C5*, *C6*, *C7*, *C8*. The size of the angle depends on the amount of rotation. If the line is rotated past *C8* into its first position *C1*, the line is said to have made a complete rotation; and the moving end of the line as it passed from point *1*, to points *2*, *3*, *4*, *5*, *6*, *7*, *8* and to *1* again is said to have made a **circumference**.

The circumference is used to measure angles that have their vertices at its center *C*.

A circumference is divided into 360 equal parts called **degrees**.

A portion of a circumference is called an **arc**.

A half of a circumference is called a **semi-circumference**; it contains 180 degrees.
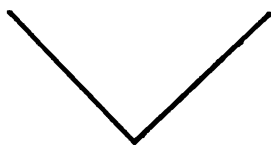
A quarter of a circumference is called a **quadrant**; it contains 90 degrees.

An angle measured by a quarter of a circumference is called a **right angle**.
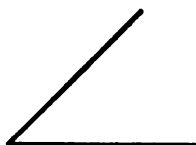
A right angle is made by one fourth of a complete rotation. "Square corner" is a child's name for a right angle.

The sides of a right angle are said to be **perpendicular** to each other.
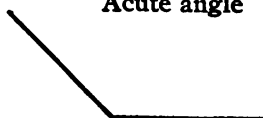
An **acute angle** is less than a right angle.

Right angle

Acute angle

Obtuse angle

An **obtuse angle** is greater than a right angle.

For fine measurements, a degree is divided into 60 equal parts called **minutes**, and a minute is subdivided into 60 **seconds**.
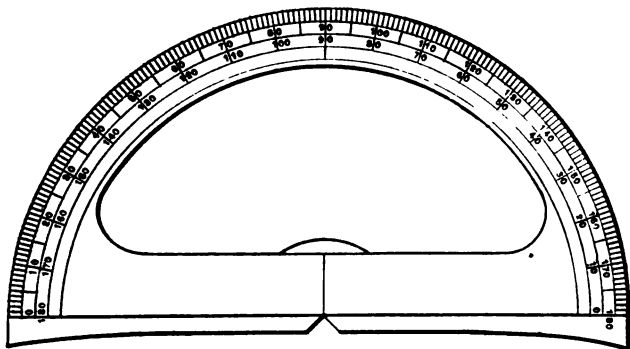
A protractor is an instrument used in measuring and constructing angles.

Table of Circular Measure
60 seconds (″) = 1 minute (′)
60 minutes   = 1 degree (°)
360 degrees   = 1 circumference
90 degrees   = 1 quadrant

NOTE TO TEACHER. The children will acquire the idea of an angle most readily by a few simple exercises in constructing and measuring angles with the aid of a protractor.

Square Measure

197. Square measure, or surface measure, is used in measuring surfaces. The standard unit of measurement is the square yard. It is a square surface one yard long and one yard wide.

Table of Square Measure	
144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30¼ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

The acre is used in measuring land. A square field 209 feet long and 209 feet wide measures about one acre.

Cubic Measure

198. Cubic measure is used in measuring solids and other volumes. The standard unit is the cubic foot. It is equal to the volume of a cube one foot in length, one foot in width, and one foot in height or depth.

A body has three measurements or dimensions: length, breadth, and thickness or height.
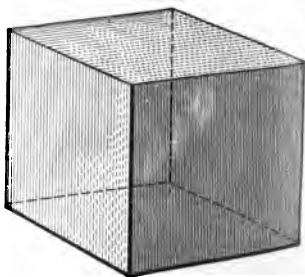
Cubic inch

A cube is a body whose length, breadth, and thickness are equal. It is bounded by six equal square surfaces.

A cubic foot is a cube each of whose dimensions is one foot.

Table of Cubic Measure	
1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet of wood	= 1 cord
231 cubic inches (of liquid)	= 1 gallon
2150.4 cubic inches (of grain, etc.)	= 1 bushel

NOTE. A cord is a pile of wood 8 ft. long, 4 ft. wide, and 4 ft. high.

Avoirdupois Weight

199. Avoirdupois weight is used in weighing grain, coal, and other bulky merchandise. The standard unit is the avoirdupois pound containing 7000 Troy grains.

Table of Avoirdupois Weight	
16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)
20 hundredweight	= 1 ton
2240 pounds	= 1 long ton

Other standard units in common use are:

196 pounds of flour	= 1 barrel (bbl.)	
200 pounds of beef	= 1 barrel	
60 pounds of wheat	= 1 bushel *	
56 pounds of corn	= 1 bushel	
56 pounds of rye	= 1 bushel	
48 pounds of barley	= 1 bushel	
60 pounds of potatoes	= 1 bushel	
32 pounds of oats	= 1 bushel	

* In some states, the legal weight of a bushel of potatoes or grain is slightly different from the weight here given.

Dry Measure

200. Dry measure is used in measuring dry articles such as grains, vegetables, and fruits. The standard unit of measurement is the bushel, containing 2150.4 cubic inches, or about $1\frac{1}{4}$ cubic feet.

Table of Dry Measure

2 pints (pt.)	= 1 quart (qt.)	
8 quarts	= 1 peck (pk.)	
4 pecks	= 1 bushel (bu.)	

Liquid Measure

201. Liquid measure is used in measuring liquids, such as water, oil, liquors, and milk. The standard unit is the gallon containing 231 cubic inches.

Table of Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
63 gallons	= 1 hogshead (hhd.)

Troy Weight

202. Troy weight is used in weighing the precious metals such as gold and silver.

Table of Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt.) or (dwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)
5760 grains	= 1 pound

203.

Miscellaneous Table

12 things	= 1 dozen (doz.)
12 dozens	= 1 gross (gro.)
12 gross	= 1 great gross
144 things	= 1 gross
20 things	= 1 score
24 sheets	= 1 quire
20 quires	= 1 ream

Time Measure

204. Time measure is used in measuring time. The standard unit of time is the day of 24 hours. It is the time required for the earth to make one complete rotation upon its axis.

The year consists of 365 days. It is the time required for the earth to make one complete revolution around the sun.

In telling the time of day, the hours before midday are called forenoon and those after midday are called afternoon. A.M. designates forenoon; P.M., afternoon; thus, 7 A.M. means 7 o'clock in the morning or forenoon; 4 P.M. means 4 o'clock in the afternoon; 12 M. indicates noon; 12 P.M. indicates midnight.

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Table of Time Measure

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
365 days	= 1 year (yr.)
366 days	= 1 leap year
100 years	= 1 century
7 days	= 1 week (wk.)
30 days	= 1 month (mo.)
12 months	= 1 year
52 weeks	= 1 year

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English Money

205. English money is the currency used in Great Britain and Ireland. The unit is the pound sterling (£), a gold coin, worth $4.8665, or about $4.87 in U. S. money. The shilling is worth a little more than 24 cents, and the penny is worth about 2 cents of our money.

Table of English Money	
4 farthings (far.)	= 1 penny (d.)
12 pence	= 1 shilling (s.)
20 shillings	= 1 pound sterling (£)
21 shillings	= 1 guinea (G.)

Other European countries use a decimal system of currency similar to our own system. The unit varies according to the country. The unit in Germany is the **mark**. Its exact value is $.238. It is divided into a hundred parts; each part is a **pfennig**.

The unit in France is the **franc**. Its exact value is $.193. It is divided into a hundred parts; each part is a **centime**.

The approximate value of the mark is $\frac{1}{4}$ of a dollar. The approximate value of the franc is $\frac{1}{5}$ of a dollar.

Review of Tables

206. TYPICAL ORAL EXERCISES

1. How many inches in 1 foot? 5 ft.? $\frac{1}{2}$ ft.? $7\frac{1}{2}$ ft.? 10 ft.? 12 ft.?

2. How many feet in 24 inches? 48 in.? 84 in.? 132 in.?

3. How many feet in 4 yards? $\frac{1}{2}$ yard?
9$\frac{1}{2}$ yards? $\frac{1}{3}$ yard?   10$\frac{1}{3}$ yards?

4. How many yards in 24 ft.?   63 ft.?   126 ft.?

5. How many yards in 2 rods? $\frac{1}{2}$ rd.?   4 rd.?

6. How many rods in $\frac{1}{2}$ mi.?   3$\frac{1}{2}$ mi.? $\frac{1}{4}$ mi.?

7. How many yards in 15 ft.?   in 10 rd.?
1 mi.? $\frac{1}{4}$ mi.? $\frac{3}{4}$ mi.?

8. How many ft. in $\frac{2}{3}$ yard? $\frac{1}{2}$ mi.? $\frac{1}{5}$ mi.?

9. How many square inches in 1 sq. ft.?
3 sq. ft.?   5 sq. ft.? $\frac{1}{2}$ sq. ft.?

10. How many square feet in 288 sq. in.?   ·
1440 sq. in.?

11. How many square yards in 18 sq. ft.?
72 sq. ft.?   288 sq. ft.?

12. How many square yards in 2 sq. rd.?
10 sq. rd.?   5 sq. rd.?

13. How many square rods in 1 acre? $\frac{1}{2}$ A.?
$\frac{1}{4}$ A.? $\frac{3}{4}$ A.?   1$\frac{3}{4}$ A.?

14. How many acres in 2 sq. mi.?   10 sq. mi.?
$\frac{1}{2}$ sq. mi.?   5 sq. mi.?

15. How many square miles in 1280 A.?
2560 A.?

16. How many cubic inches in 1 ft.?   2 ft.?
$\frac{1}{2}$ ft.? $\frac{1}{4}$ ft.? $\frac{3}{4}$ ft.?

17. How many cubic feet in 1 cu. yd.?
5 cu. yd.? $\frac{1}{3}$ cu. yd.?

REDUCTION

207. Reducing a denominate number to units of smaller denomination is called **Reduction Descending**; *e.g.* 6 feet may be reduced to 72 inches.

208. **Reduction Ascending** is the reduction of a denominate number to units of larger denomination; *e.g.*, 6 feet may be reduced to 2 yards.

Reduction Descending

209. Reduce 4 yd. 2 ft. to inches.

Short Process

Solution

4 yd. = 4 × 3 ft. = 12 ft.
12 ft. + 2 ft. = 14 ft. *Ans.*

$$\begin{array}{r} 4 \\ 3 \\ \hline 12 \\ 2 \\ \hline 14 \text{ ft. } Ans. \end{array}$$

Reduce 6 bu. 1 pk. 3 qt. to quarts.

Short Process

Solution

6 bu. = 6 × 4 pk. = 24 pk.
24 pk. + 1 pk. = 25 pk.
25 pk. = 25 × 8 qt. = 200 qt.
200 qt. + 3 qt. = 203 qt. *Ans.*

$$\begin{array}{r} 6 \\ 4 \\ \hline 24 \\ 1 \\ \hline 25 \\ 8 \\ \hline 200 \\ 3 \\ \hline 203 \text{ qt. } Ans. \end{array}$$

WRITTEN EXERCISES

210. 1. Reduce 75 bu. 3 qt. to quarts.*

2. Change 3 da. 15 hr. to minutes.

3. How many grains in 17 dwt. 19 gr.?

4. How many pounds in 476 T. of coal?

5. How many grains in 25 oz. (avoir.)?

6. How many square inches in 45 sq. ft.?

7. To how many seconds are 876 min. equal?

8. 25 bbl. and 60 lb. of beef equal how many pounds?

9. Reduce 1 mi. 47 rd. to yards

10. Make up 3 original examples in reduction descending based on each table that you have studied.

11. How many ounce packages of chocolate can be made from 37 lb.?

12. A coffee merchant puts up 1-lb. packages of coffee. How many packages does he make up from 6 bags of coffee, each bag weighing 56 lbs.?

13. How many pieces measuring 12 sq. ft. can be cut from 3 rolls of oilcloth each containing 36 sq. yd.?

14. How many quart bottles can be filled from 22 bbl. of vinegar?

---

* Note to Teacher. After the succession of steps is understood, the multiplier may be omitted. Sufficient drill should be given to make the process a matter of habit.

Reduction Ascending

211. Reduce 5670 oz. (avoirdupois) to pounds.

Process	Explanation
16)5670	16 oz. = 1 lb.
354⅜ lb.	5670 oz. = ⁵⁶⁷⁰⁄₁₆ lb. = 354⅜ lb.

Reduce 442 qt. to higher denominations.

Process	Explanation
8)442	8 qt. = 1 pk.
4) 55 pk. 2 qt.	442 qt. = ⁴⁴²⁄₈ pk. = 55 pk. 2 qt.
13 bu. 3 pk.	4 pk. = 1 bu.
	55 pk. = ⁵⁵⁄₄ bu. = 13 bu. 3 pk.
13 bu. 3 pk. 2 qt. *Ans.*	∴ 442 qt. = 13 bu. 3 pk. 2 qt.

WRITTEN EXAMPLES

1. Reduce 4579 ft. to yards.

2. Change 457 oz. to avoirdupois pounds.

3. Reduce 786 Troy gr. to ounces.

4. Change 1894 rd. to miles.

5. How many bushels in 53,760 cu. in.?

6. Reduce 16,549 sq. rd. to acres.

7. How many pecks and bushels in 985 qt.?

8. Change 560 qt. to barrels.

9. How many days and hours in 7864 min.?

10. Make up 3 original examples in reduction ascending based on each table that you have studied.

Q

212. 1. A barrel holds 365 lb. of white lead; 6 lb. of white lead will fill a quart can. How many quart cans can be filled from the contents of the barrel?

2. How many packages of tea, each weighing 1 lb., can be made up from 2800 oz. of tea?

3. Two barrels of wine are emptied into quart bottles. How many bottles are required?

4. How many boxes will be required to pack 1728 dolls, if each box holds one dozen dolls?

5. A printer prints 86,000 sheets which he packs into reams; how many reams is he able to make up? How many quires of paper are left?

6. An ounce of shellac is needed for 1 sq. yd. of oilcloth; how many pounds must be purchased to shellac 576 sq. ft. of cloth?

7. How many bushels of seed in 2764 qt.?

8. An order is given for 4244 bronze medals, each medal weighing 1 oz. How much bronze (hundredweight and pounds) will be required to fill the order?

Denominate Fractions

213. 1. What part of a ton is 50 lb.?

$$2000 \text{ lb.} = 1 \text{ T.}$$
$$50 \text{ lb.} = \tfrac{50}{2000} = \tfrac{1}{40} \text{ T.}$$

2. What part of a bushel is $\frac{1}{2}$ pk. ?

$$4 \text{ pk.} = 1 \text{ bu.}$$
$$\tfrac{1}{2} \text{ pk.} = \frac{\frac{1}{2}}{4} = \frac{1}{8} \text{ bu.}$$

3. What part of a bushel is $2\frac{1}{4}$ pk. ?

$$4 \text{ pk.} = 1 \text{ bu.}$$
$$2\tfrac{1}{4} \text{ pk.} = \frac{2\frac{1}{4}}{4} = \frac{9}{16} \text{ bu.}$$

4. What decimal part of a day is 3 hr. ?

$$24 \text{ hr.} = 1 \text{ da.}$$
$$3 \text{ hr.} = \tfrac{3}{24} = \tfrac{1}{8} = .125 \text{ da.}$$

5. Reduce £ $\frac{7}{8}$ to lower denominations.

$$£1 = 20 \, s.$$
$$£\frac{7}{8} = \frac{7}{\underset{2}{\cancel{8}}} \text{ of } \overset{5}{\cancel{20}}s. = \frac{35}{2} = 17\tfrac{1}{2}s.$$
$$1 s. = 12d.$$
$$\tfrac{1}{2}s. = \frac{1}{\cancel{2}} \text{ of } \overset{6}{\cancel{12}}d. = 6d.$$
$$£\tfrac{7}{8} = 17s. \; 6d. \quad \textit{Ans.}$$

ORAL EXERCISES

1. What part of a ton is 100 lb. ?

2. What part of a mile is 60 rd. ?

3. 3 qt. equals what part of a bushel ?

4. What part of a ream is 12 quires ?

5. What part of a gallon is 2 qt. ?

6. What part of a gallon is 2 qt. 1 pt. ?

7. What part of a peck is 2 qt. 1 pt. ?

8. What part of a gross is $\frac{3}{4}$ doz. ?

9. What part of a right angle is an angle of 45 degrees?

10. What part of a yard is 1 ft. 6 in. ?

11. What decimal part of a year is 4 mo. ?

12. What decimal part of a bushel is 4 qt. ?

13. What decimal part of a mile is 160 rd. ?

14. What decimal part of a hundredweight is $7\frac{1}{2}$ lb. ?

15. What decimal part of a square foot is 72 sq. in. ?

16. How many feet in $\frac{1}{4}$ mi. ?

17. How many pounds in $\frac{3}{4}$ T. ?

18. Reduce $\frac{7}{8}$ gal. to quarts.

19. How many degrees in $\frac{3}{5}$ of a right angle?

20. $\frac{7}{16}$ of a ream of paper equals how many sheets ?

21. How many grains in $\frac{5}{8}$ oz. (Troy) ?

WRITTEN EXERCISES

214. 1. What part of an ounce (Troy) is 240 gr. ?

2. What part of a gross is 18 pens ?

3. What part of a pound (avoirdupois) is $\frac{3}{4}$ oz. ?

4. What part of 3 lb. is 5 oz. (Troy) ?

5. What part of 5 T. is 50 cwt. ?

6. What part of a bushel is 2½ pk. ?

7. What decimal part of a square foot is 100 sq. in. ?

8. What decimal part of a gallon of water is 77 cu. in. ?

MEASUREMENT OF SURFACES

For Reading and Discussion

215. Name 5 objects with flat or plane surfaces. Name 5 objects with curved or round surfaces.

Which is the larger; the surface of your desk or the surface of the teacher's desk ?

Which is the smaller, the surface of the black-board or the surface of the ceiling ?

Using a sheet of paper as a measuring unit, find how many times its surface is contained in the surface of your desk.

A plane surface has two linear dimensions, length and width.

The **area** or size of a surface is usually expressed in the units of square measure: *i.e.* square inch, square yard, acre, square mile.

The circle, the triangle, the quadrilateral, the parallelogram, and the rectangle are a few of the many plane figures that may be drawn on a plane surface.

Rectangles

(Each pupil should have a ruler and a right triangle.)

D _____ C

A _____ B
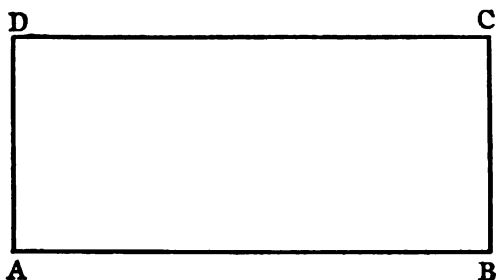
RECTANGLE

(length 2½ in., width 1¼ in.)

216. The figure *ABCD* is a rectangle.

A **rectangle** is a plane figure bounded by four straight lines and having four right angles. (See Art. 196.)

Name the sides of the rectangle. Measure the sides. How long is each side?

Name the right angles in the rectangle.

The distance from *A* to *B* is called the **length** of the rectangle.

The distance from *A* to *D* is called the **width** of the rectangle.

Draw rectangles having the following dimensions:

	LENGTH	WIDTH		LENGTH	WIDTH
1.	2 in.	4 in.	**3.**	5 in.	3 in.
2.	4 in.	3 in.	**4.**	4 in.	2 in.

If the length and width of a rectangle are equal, the figure is called a square.

Draw a square each side being 2 inches.

Draw a square each side being 3 inches.

Draw a square each side being 4 inches.
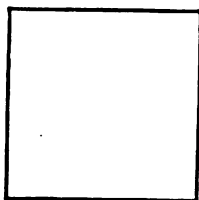
ONE-INCH SQUARE

A **square** is a rectangle having all sides equal.

To Find the Area of a Rectangle
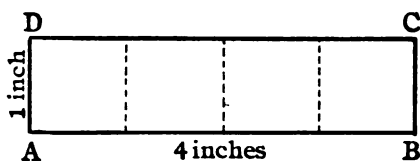
217. Cut out of paper a rectangle 4 inches long and 1 inch wide.

Fold the rectangle into 4 equal parts (indicated by the dotted lines).

Each of the small figures is a square. Why?

What is the length of each of the squares?

What is the width of each of the squares?

Therefore each square is an **inch square**, and its area is 1 square inch. Hence the area of the rectangle is 4 square inches.

Cut out of paper a rectangle 2 inches wide and 4 inches long.

Divide the rectangle into two equal parts by folding, as indicated by the dotted line.

The rectangle is now divided into how many equal rectangles? What is the length of each rectangle? What is the width?
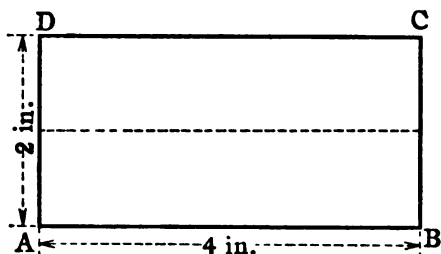
The area of one of these rectangles is how many square inches?

Therefore the area of both rectangles is how many square inches?

In a similar way find the area of each of the following rectangles, which are drawn to scale $\frac{1}{4}$:
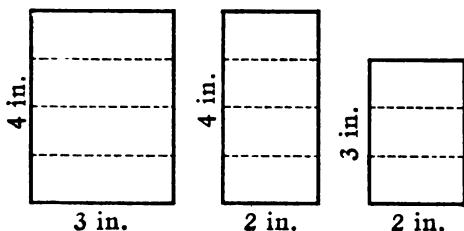
Find the number of smaller rectangles (or strips); multiply the area of 1 strip by the number of strips in the rectangle.

218. RULE. **The area of a rectangle is equal to the product of the length by the width expressed in like units:**

e.g. the area of a rectangle that is 8 in. long and 6 in. wide is 48 sq. in.

The Area of a Right Triangle

219. Cut out of paper a rectangle 4 in. long and 3 in. wide. Fold and cut the rectangle into two parts along the dotted lines.
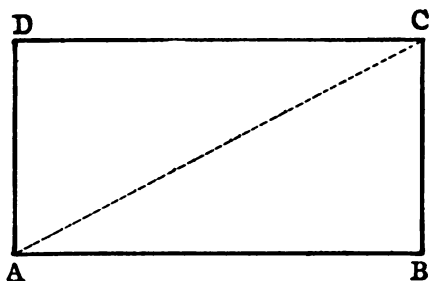
This divides the rectangle into two triangles. Since each triangle has a right angle in it, each triangle is called a **right triangle**.

How do the two triangles compare in size? How do you know?

Therefore, the area of each triangle is what part of the area of the rectangle?

What is the area of the rectangle? What is the area of each triangle?

How may the area of any right triangle be found?

In the triangle *ABC*, the side *AB* on which the

triangle stands is called the **base**; and the side BC is called the **height** or **altitude**.

220. RULE. **The area of a right triangle is equal to one half the product of the base by the altitude expressed in like units:**

e.g. if the length or base is 10 ft. and the height or altitude is 8 ft., the area of the triangle is $\frac{1}{2}$ of (10 × 8) sq. ft. or 40 sq. ft.

To Find the Area of a Parallelogram

221. When two lines, like AB and CD, remain the same distance apart, the lines are **parallel**.

C ———————————— D

A ———————————— B

Railroad tracks are a good illustration of parallel lines.

In the figure $ABCD$, the lines AB and DC are parallel; the lines AD and BC are parallel.
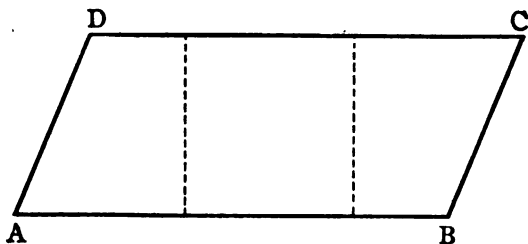
The figure $ABCD$ is a **parallelogram**.
The line AB is the **base** of the parallelogram.
The dotted lines represent the **height** or **altitude**

of the parallelogram. (They are perpendicular to
the base AB.)

Cut out of paper a parallelogram having a base
of 4 inches and an altitude of 3 inches. Cut off
the shaded part.

Place this part at CB.
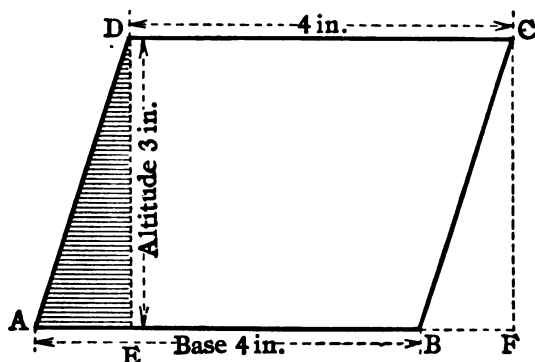
What kind of figure is formed?

What line is its base or length? How long is
the base?

What line is its width or height?

How does the new figure compare with the old
figure in area?

Therefore, how may you find the area of a
parallelogram?

222. RULE. **The area of a parallelogram is equal to
the product of the base by the altitude expressed in like
units:**

e.g. if the base of a parallelogram is 7 ft. and the

altitude is 3 ft., the area of the parallelogram is 21 sq. ft.

To Find the Area of a Triangle

223. In Art. 219 we learned how to find the area of a right triangle. We shall now learn how to find the area of any kind of triangle.

Draw these parallelograms:
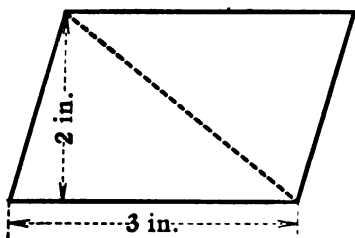
Cut each parallelogram out of paper.

Fold and cut each parallelogram along the dotted line.
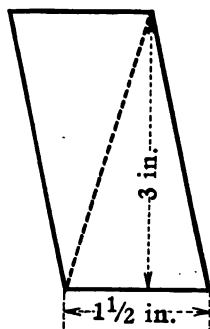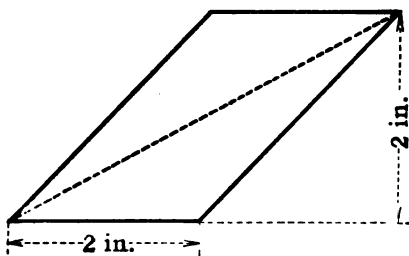
How do the two triangles formed from the first parallelogram compare in size? What are the dimensions of each of the triangles? What is the area of each triangle?

224. RULE. **The area of a triangle is equal to one half the product of the base by the altitude:**

e.g. if the base of a triangle is 9 ft. and the altitude is 3 ft., the area is $13\frac{1}{2}$ sq. ft.

WRITTEN PROBLEMS

225. **1.** Find the area of a square whose side is 8 in.

2. What is the area of a 12-in. square?

3. What is the area of a rectangle having a length of 5 ft. and a width of 1 ft. 8 in.?

4. How many square yards in a rectangular field, length 125 ft., breadth 25½ ft.?

5. The dimensions of your classroom are 24′ 6″ and 18′ 8″.* What is the floor surface?

6. The base of a rectangular baseball field is 75 yd. 2 ft., and the altitude is 125 ft. What is its area in square yards?

7. How many acres are there in a rectangular field 75 rd. long and 35 rd. wide?

8. What is the cost of sodding a rectangular garden 84 ft. long and 45 yd. wide, at 25 ct. per square foot?

9. Which is the larger, a square field whose length is 55 rd., or a rectangular field, 67 rd. × 36 rd.? By how much?

10. The area of a rectangular field is 100 A. The length is 250 rd. What is the width?

11. The area of your playground is ½ A. The length is 120 ft. What is its width?

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* Mechanics frequently express feet by ′ and inches by ″; *e.g.* 5 ft. 8 in. may be written 5′ 8″.

12. A stationer buys 1000 sheets of bond paper 22 in. × 18 in., of which he makes pads, each containing 80 sheets 6 in. × 9 in. How many pads will he have?

13. What will a landlord have to pay for tarring the roof of a house 25 ft. × 75 ft. at 18 ct. per square foot?

14. An oblong baseball field contains 25000 sq. ft. The length is 41⅔ yd. What is the width?

Find the missing quantity of the following rectangles:

	LENGTH	WIDTH	AREA
15.	12 ft. 6 in.	——	240 sq. ft.
16.	85 rd.	——	21.25 A.
17.	9 yd.	——	252 sq. yd.
18.	——	510 ft.	340 sq. rd.

EXERCISE

226. Find the missing quantity of the following parallelograms:

	BASE	ALTITUDE	AREA
1.	12 ft.	4 ft. 8 in.	?
2.	?	16 yd.	2 sq. rd.
3.	18 rd.	24 yd.	?
4.	265 rd.	?	1½ sq mi.
5.	1500 ft.	1200 yd.	? sq. mi.
6.	750 rd.	450 rd.	? A.
7.	160 rd.	?	50 A.

EXERCISES

227. 1. Find the area of a triangle whose base is 10 ft. and whose altitude is 16 ft.

2. How many square yards in a triangular field the base of which is 240 yd. and the altitude $\frac{1}{2}$ as much?

3. Which is the larger, a triangular field whose base and altitude are 250 yd. and 124 yd. respectively, or a rectangular field 1200 ft. by 120 ft.? By how much?

4. The triangular park at the intersection of two streets has a base of 20 ft. and an altitude of 75 ft. How many square yards in the park?

Find the areas of the triangles whose dimensions are given below:

BASE	ALTITUDE	BASE	ALTITUDE
5. 10 in.	5 in.	**8.** 15 rd.	$3\frac{1}{2}$ rd.
6. 3 ft. 4 in.	$2\frac{1}{2}$ ft.	**9.** 250 rd.	450 rd.
7. 3 ft.	2 ft. 2 in.	**10.** 9 yd.	11 yd. 2 ft.

Miscellaneous Problems in Denominate Numbers

WRITTEN EXERCISES

228. 1. A merchant tailor is told that a certain kind of cloth sells at $2.35 per yard, in single yards, and at $1.85 per yard when bought by the bolt. How much does he save by buying an entire bolt of cloth measuring $36\frac{1}{2}$ yd.?

2. What will be the charge for plastering the walls of a room measuring 5236 sq. ft. at the rate of 30 ct. per square yard?

3. How many cubic feet in 12,675 cu. in.?

4. A man works 8 hr. a day. How many hours will he work in 7 wk., excluding Sundays?

5. 1 bu. of wheat weighs 60 lb. What is the weight of a quart?

6. At the rate of 9 ct. for 2 qt. of milk, what is the cost of 12 gal.?

7. A steamship on the Great Lakes carries a cargo of 25,000 bu. of wheat. How many pounds in the cargo?

8. How many spoons, each weighing $\frac{3}{4}$ oz., can be made from 1 lb. 2 oz. gold?

9. How many years in four score and ten?

10. A box holds 25 bu. of wheat; what is the capacity of the bin in cubic inches?

11. What will 3 pk. of potatoes cost at $1.90 per bushel?

12. How many $2\frac{1}{2}$-lb. bags of salt can be filled from a box weighing $\frac{1}{8}$ of a ton?

13. A man digs a trench that contains 236 cu. yd. at the rate of $12\frac{1}{2}$ ct. per cubic foot. How much does he receive for his work?

14. A boy attended school 5 hr. each day for 20 wk. If there were 7 holidays, how many hours did he spend in school?

15. A bag of peanuts holding 1½ bu. is bought for $1.65. The peanuts are roasted and sold at the rate of 5 ct. per pint. What is the profit?

16. Your father finds that 5 strips of carpet each 5½ yd. long are needed to carpet the parlor of your home. What is the amount of his purchase if he buys the carpet at $1.87½ per yard?

17. A boy runs a furlong in 7¾ min.; at that rate how long will it take him to run a mile and a half?

18. Which is the cheaper way to buy books, at 85 ct. per gross or at 9 ct. per doz.? How much would be saved by buying ½ great gross at the cheaper rate?

19. At the rate of 2 for 5 ct., what is the cost of 6 doz. pencils?

20. How much will be received for a gallon of molasses which is sold at 8 ct. per pint?

21. A small pitcher holds ½ pt. of cream; how many times can this pitcher be filled from a 2-qt. measure of cream? From a gallon measure?

22. To cover the floor of a room 650 ft. of lumber is required. At the rate of $24.50 per thousand feet, what is the lumber worth?

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23. 196 lb. of wheat flour make one barrel. What will be the weight of a cargo of 1200 bbl.? What will be the freight charges at $18\frac{3}{4}$ ct. a barrel?

24. A running track has 8 laps to the mile. Two boys run a lap in $6\frac{1}{2}$ min. and $7\frac{3}{4}$ min. respectively. At the same rates, how much longer will it take one to run a mile than the other?

25. A gang of laborers build 1 furlong of railroad in $3\frac{1}{2}$ wk.; how long will it take them to build a section $25\frac{1}{2}$ mi. long?

26. A box containing a gross of screws is bought for $.65; the screws are sold at the rate of 3 for 10 ct. What is the amount gained on the sale?

27. A tile layer receives $17.50 for laying one thousand tiles, each tile measuring $\frac{1}{2}$ sq. ft. How much will he receive if the surface to be tiled measures 250 sq. yd.?

28. A relay team of 4 boys wins gold medals each containing 1 oz. 6 dwt. of gold. What is the value of the medals if the gold costs $1.10 per pennyweight and the die $12.75?

29. The cost of labor in building a wall that requires 4500 cu. ft. of cement is 95 ct. per cubic yard. The cement cost $347. What is the total cost?

30. Which is cheaper, to buy coal at the rate of 40 ct. a hundredweight or to buy it at $6.50 per

ton? How much would be saved by buying 15 T. at the cheaper rate?

31. The distance between New York and Liverpool is 2800 miles. The *Mauretania* sails that distance in $4\frac{3}{4}$ da. At what rate does she sail per day? Per hour?

32. The Chicago Limited train covers the 900 miles between Chicago and New York in $\frac{3}{4}$ da. What is its average speed?

33. How many cubic inches in a cistern whose capacity is $50\frac{2}{3}$ gal.?

34. A painter bought $25\frac{3}{4}$ gal. of varnish at the rate of 12 ct. per quart. How much did he have to pay?

35. The gold medal offered as first prize in school weighs 2 oz.; a silver medal weighs $1\frac{3}{4}$ oz. The cost of the metals are $1.10 and 45 ct. per pennyweight respectively. What is the difference in value between the first and the second prize?

36. A barrel is filled with vinegar. At the rate of 5 ct. per quart, what will the barrel of vinegar be worth?

37. There are 52 classes in a school building. Each class requires $1\frac{1}{2}$ pt. of ink each week. How much ink is required for the entire school in 8 weeks? Give the answer in gallons.

38. A grocer buys a barrel of flour for $5.75. This he sells in 3½-lb. packages at 12 ct. each. How much does he gain?

39. Which is cheaper, paying a painter at the rate of 9½ ct. per square foot for painting 450 sq. yd. of wall space or paying him $400 for the entire work? How much is saved by taking the cheaper way?

40. An automobile runs at the rate of 33 mi. per hour; what distance does it cover in ⅜ da.?

41. 20 city blocks equal 1 mi.; what is the average length of each block in yards? In feet?

42. At the rate of $25 per 10 dwt. what is the cost of 12 doz. watch cases, each weighing 5½ dwt.?

43. A goldsmith receives an order to make 3 loving-cups, each containing 5 oz. 6 dwt. of silver and 2 oz. 4 dwt. of gold. What is the weight of the three loving-cups?

44. What is the weight of a dozen silver spoons each weighing 1 oz. 15 dwt.?

45. A farmer raises 25 bu. of wheat and 10 bu. of corn on 1 A. How many bushels of grain can be raised, at that rate, on a similar farm containing ¾ sq. mi.?

46. A man bought 5 A. of land for $2275. He laid it out in lots which he sold at $125 each. If each acre was divided into 8 lots, how much did he make on the transaction?

47. Your mother pays 6 ct. per quart for potatoes. How much would she save by buying a bushel for $1.50?

48. Which is the cheaper way to buy walnuts, at 4 ct. per pint or at 60 ct. per bushel? How much will be saved by buying 1 bu. at the cheaper rate?

49. A grocer bought 6 bbl. of Baldwin apples, each barrel containing $2\frac{1}{2}$ bu., at $3.30 per barrel. These he sold at 35 ct. per peck. How much did he gain or lose?

50. A stationer bought a ream of note paper for $3, and sold it at 20 ct. per quire. What was his profit?

51. There are forty boys in a class. If each boy receives 4 sheets of paper during the day, how many quires will be required in a week?

52. The quantity of water running from a faucet in a school is $1\frac{1}{2}$ qt. a minute. There are 10 faucets. If the faucets are kept open for 2 minutes, how much water runs out?

53. The distance between New York and Chicago is 900 mi. A train that travels at the rate of 45 mi. an hour leaves New York at 4 o'clock P.M. When does it reach Chicago? (New York time.)

54. A cubic foot of water weighs $62\frac{1}{2}$ lb. What is the weight of the water in a tank holding 25 gal.?

55. There are four floors to a school; each floor contains 15 rooms; the average area of each room is 450 sq. ft. What is the total floor area of the building, if the area of the hallways equals 1465 sq. ft.?

56. A crown contains 10 oz. 10 dwt. gold, and 5 oz. 15 dwt. silver. The gold is worth $1.06 per pennyweight, and the silver 2¾ ct. per pennyweight. How much more is the gold in the crown worth than the silver? What is the value of the metal in the crown, not counting the cost of labor?

57. 18 horses use 2 bu. 6 pk. of oats daily. What is the amount used in a week? At $1.25 per bushel, what is the cost of feeding each horse per day? Per week?

58. A grocer sells cleaned rice in 1-qt. tin boxes. How many of these boxes will he be able to fill from a bag holding 6¾ pecks of rice?

59. Two men begin work on Monday morning at 8 o'clock. One is paid at the rate of $15 per week; the other, at the rate of 35 cents per hour. They work from 8 A.M. until 5 P.M. for 6 days each week.

(*a*) How much will each receive at the end of 30 da.?

(*b*) Which is the more profitable way of working, by the week or by the hour, at the above rates?

60. How many pounds of cotton are required to make 30,000 yd. of a certain cotton cloth, 3 lb. of cotton being required for 2 yd. of the cotton cloth?

Draw to scale and find the area of the following:

61. The surface of your desk. (Scale $\frac{1}{8}$.)

62. The floor of your classroom, obtaining the dimensions from your teacher. (Scale $\frac{1}{8}$ in. to 1 ft.)

63. The panes of glass in the lower sash of one window. (Scale $\frac{1}{4}$ in. to 1 ft.)

64. The entire blackboard surface of your classroom. (Scale $\frac{1}{16}$ in. to 1 ft.)

65. Make a map of a city block 800 ft. by 200 ft. divided into city lots 100 ft. by 25 ft. (Scale 1 in. to 100 ft.)

PERCENTAGE

229. You have already learned to work with common fractions and with decimals, and you know that the results have the same value whether the numbers used are common fractions or decimal fractions. For example, we may find one half of $147.60 either by dividing by 2 or by multiplying by .5. In some cases, we may shorten the work by changing a given decimal to a common fraction. In some cases, however, it is well to reduce the given common fraction to its decimal form. Then instead of dealing with fractions having all sorts of denominators, we have to deal only with denominators of 10 or some power of 10, which we can express in decimal notation the same as whole numbers. We may simplify matters still more by reducing all the decimal fractions to one denominator. In business life, one seldom hears of any common fractions except $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{8}$, and their multiples $\frac{3}{4}$, $\frac{3}{8}$, etc. Nor does one often hear of the decimal fractions, tenths, thousandths, ten-thousandths, etc. The general custom is to use

hundredths, to which is given the special name **per cent**. Thus, the teacher may say that the class got 95 per cent in spelling, which is high, or only 50 per cent in grammar, which is rather low. The mark in spelling means that out of every 100 words spelled, 95 were spelled correctly and 5 were not spelled correctly; and the mark in grammar means that half the answers were right and half were not right.

Per cent means hundredths.

Per cent may be written as a common fraction, or as a decimal fraction, or by means of the per cent sign; *e.g.* 1 per cent may be written $\frac{1}{100}$, or .01, or 1 %.

Notice that the per cent sign takes the place of the denominator 100 in the common fraction, and of two decimal places in the decimal fraction.

Changing Common Fractions and Decimal Fractions to Per Cents

ORAL EXERCISES

230. Read the following: first, as printed; second, as per cent; *e.g.* "5 hundredths or 5 per cent."

1. $\frac{5}{100}$ $\frac{8}{100}$ $\frac{12}{100}$ $\frac{50}{100}$ $\frac{65}{100}$ $\frac{42}{100}$

2. .05 .08 .15 .40 .78 .31

3. $.12\frac{1}{2}$ $.33\frac{1}{3}$ $.14\frac{2}{7}$ $.02\frac{1}{2}$ $.18\frac{3}{4}$ $.16\frac{2}{3}$

4. $\dfrac{3\frac{1}{3}}{100}$ $\quad$ $\dfrac{8\frac{1}{3}}{100}$ $\quad$ $\dfrac{62\frac{1}{2}}{100}$ $\quad$ $\dfrac{22\frac{2}{9}}{100}$ $\quad$ $\dfrac{27\frac{3}{5}}{100}$ $\quad$ $\dfrac{12\frac{15}{16}}{100}$

HINT. The following must be reduced to hundredths or per cent; *e.g.* $.125 = .12\frac{1}{2}$ or $12\frac{1}{2}\%$.

5. .125 $\quad$ .375 $\quad$ .625 $\quad$ .875 $\quad$ .500 $\quad$ .5

6. .2 $\quad$ .1 $\quad$ .8 $\quad$ .600 $\quad$ .30 $\quad$ .4

7. .315 $\quad$ .3125 $\quad$ .0175 $\quad$ .235 $\quad$ .0425 $\quad$ .1875

8. 1.25 $\quad$ 2.50 $\quad$ 3.75 $\quad$ 10.25 $\quad$ 1.10 $\quad$ 1.04

9. 5.00 $\quad$ 1.00 $\quad$ 1. $\quad$ 2. $\quad$ 5. $\quad$ 10.

10. 6.25 $\quad$ 6. $\quad$ 6.5 $\quad$ 1.5 $\quad$ 6.025 $\quad$ 1.1875

11. $.00\frac{3}{4}$ $\quad$ .005 $\quad$ .0075 $\quad$ $.00\frac{3}{8}$ $\quad$ .00625 $\quad$ .0025

HINT. $.00\frac{3}{4}$ is read "three fourths of 1%," or "three fourths per cent."

12. $.00\frac{1}{8}$ $\quad$ $1.01\frac{1}{4}$ $\quad$ 1.0175 $\quad$ $.00\frac{2}{3}$ $\quad$ 5.105 $\quad$ 2.0325

HINT. The following must first be reduced to hundredths and then changed to per cent; *e.g.* $\frac{3}{8} = .37\frac{1}{2}$ or $37\frac{1}{2}\%$.

13. $\frac{3}{8}$ $\quad$ $\frac{1}{4}$ $\qquad$ 16. $\frac{1}{25}$ $\quad$ $\frac{9}{10}$ $\qquad$ 19. $\frac{2}{3}$ $\quad$ $\frac{1}{10}$

14. $\frac{1}{5}$ $\quad$ $\frac{7}{8}$ $\qquad$ 17. $\frac{3}{4}$ $\quad$ $\frac{5}{8}$ $\qquad$ 20. $\frac{3}{10}$ $\quad$ $\frac{3}{20}$

15. $\frac{7}{10}$ $\quad$ $\frac{1}{20}$ $\qquad$ 18. $\frac{4}{5}$ $\quad$ $\frac{1}{8}$ $\qquad$ 21. $\frac{1}{2}$ $\quad$ $\frac{1}{6}$

WRITTEN EXERCISES

231. Write all the numbers in the last exercise as per cents with the sign $\%$.

Changing Per Cents to Decimal Fractions and to Common Fractions

232. Read the following, first, as *per cent;* second, as *hundredths :*

e.g. 5 % is read, " 5 per cent, or 5 hundredths."

1.	5 %	10 %	25 %	20 %	50 %	75 %
2.	40 %	60%	80 %	30 %	70 %	90 %
3.	$33\frac{1}{3}$%	$66\frac{2}{3}$%	$12\frac{1}{2}$%	$37\frac{1}{2}$%	$62\frac{1}{2}$%	87 %
4.	$16\frac{2}{3}$%	$8\frac{1}{3}$%	$2\frac{1}{2}$%	100 %	200 %	125 %

233. *A.* Write all the per cents in the last exercise as decimal fractions.

B. Write all the per cents in the last exercise as common fractions, reduced to their lowest terms.

234. 1. Express decimally as hundredths : 25 % ; 5 % ; $2\frac{1}{2}$ % ; $\frac{1}{8}$% ; 150 % ; 300 %.

$$25\% = .25 \qquad 2\frac{1}{2}\% = .02\frac{1}{2}$$
$$5\% = .05 \qquad \frac{1}{8}\% = .00\frac{1}{8}$$
$$150\% = 1.50 \qquad 300\% = 3.00$$

2. 18 %   **3.** 54 %   **4.** 6 %

5. 103 %   **6.** $62\frac{1}{2}$ %   **7.** $16\frac{3}{4}$ %

8. $231\frac{1}{4}\%$ 9. $6\frac{1}{4}\%$ 10. $33\frac{1}{3}\%$

11. $416\frac{2}{3}\%$ 12. $8\frac{1}{3}\%$ 13. $155\frac{5}{9}\%$

14. Express as pure decimals: $12\frac{1}{2}\%$; $31\frac{1}{4}\%$; $2\frac{2}{5}\%$; $1\frac{1}{8}\%$.

$$12\frac{1}{2}\% = .125 \qquad\qquad 2\frac{2}{5}\% = .024$$
$$31\frac{1}{4}\% = .3125 \qquad\qquad 1\frac{1}{8}\% = .01125$$

15. $62\frac{1}{2}\%$ 16. $87\frac{1}{2}\%$ 17. $18\frac{3}{4}\%$ 18. $6\frac{1}{4}\%$

19. $14\frac{2}{7}\%$ 20. $1\frac{3}{8}\%$ 21. $\frac{1}{2}\%$ 22. $\frac{7}{8}\%$

23. Express as common fractions: 25%; 60%; $12\frac{1}{2}\%$; $83\frac{1}{3}\%$.

$$25\% = \frac{25}{100} = \frac{1}{4} \qquad\qquad 12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$$
$$60\% = \frac{60}{100} = \frac{3}{5} \qquad\qquad 83\frac{1}{3}\% = \frac{83\frac{1}{3}}{100} = \frac{250}{300} = \frac{5}{6}$$

24. 30% 25. 70% 26. 75% 27. $38\frac{1}{3}\%$

28. $37\frac{1}{2}\%$ 29. $87\frac{1}{2}\%$ 30. $66\frac{2}{3}\%$ 31. 50%

32. $16\frac{2}{3}\%$ 33. $62\frac{1}{2}\%$ 34. 5% 35. $2\frac{1}{2}\%$

36. 125% 37. 250% 38. $133\frac{1}{3}\%$ 39. 150%

Finding a Per Cent of a Number

ORAL EXERCISES

235. 1. How much is $\frac{1}{2}$ of 10? $\frac{1}{3}$ of 18? $\frac{2}{3}$ of 12? $\frac{7}{8}$ of 40?

2. Find $\frac{1}{100}$ of 200; $\frac{3}{100}$ of 200; $\frac{25}{100}$ of 400; $\frac{3}{100}$ of 600.

3. Find .01 of 300; .05 of 300; .08 of 300; .12 of 300.

4. How many hundredths of a number is 7 % of it ?

5. Find 7 % of 200. Of 300. Of 400. Of 800

6. Find 6 % of 500. Of 800. Of 900. Of 200.

7. Find 1 % of 500. 8 % of 500. 12 % of 500. 6 % of 500.

8. How much is 10 % of $400? 12 % of $200? 25 % of $ 200 ?

9. Think of 1 % of 400. Find 2 % ; 3 % ; 4 % ; 6 % ; 10 %.

10. Think of 1 % of 900. Find 2 % ; 4 % ; 5 % ; 7 % ; 8 %.

WRITTEN EXERCISES

236. How much is 68 % of $ 320.50?

$ 320.50
  .68
───────
256400
192300
───────
$ 217.9400

Explanation

68 % of $ 320.50 = .68 × $ 320.50.

Find 4 % and 27 % and 56 % and 88 % of the quantities given in line **1**; then, of those in line **2**; and so on :

1. $120	$ 450	$80	$ 25
2. $4000	$1250	$ 38	$ 7

3. $21.50 $16.08 $2.10 $10.05

4. 21,615 ft.   264.5 ft.   196 lb.   17.65 cwt.

5. 12.625 mi.   12,608 sq. yd.   1728 cu. in.   5760 gr.

WRITTEN PROBLEMS

237. 1. A tank contained 250 gallons of water. 28 % of the amount was drawn off.

(a) How many gallons were drawn off?

(b) How many gallons were left in the tank?

2. 350 persons were employed in a department store. 18 % were children; 62 % were women.

(a) How many children were employed?

(b) How many women were employed?

3. A plot of ground is 540 ft. long. The width is 85 % of the length. Find the width.

4. A club played 150 games and won 8 % of them. How many games were won?

5. In a certain library there are 120,000 books, of which 5 % are new books. How many new books are in the library?

HINT. 1 % of 120,000 = 1200; 5 % = ?

ORAL EXERCISES

238. Whenever the given number ends in ciphers, the process may be shortened by first finding 1 % of the given number (by striking out two

ciphers) and then finding the given per cent by multiplying.

1. Find 12 % of 400.

Explanation.   1 % of 400 is 4; 12 % of 400 is 12 times 4, or 48.

2. Find 3 % of 200 ; 500 ; 800 ; 1500 ; 2000.

3. Find 8 % of 300 ; 700 ; 1000 ; 12,000 ; 50,000.

4. Find 11 % of 400 ; 600 ; 2000 ; 1100 ; 60,000.

5. Find 15 % of 200 ; 4000 ; 30,000 ; 2000 ; 10,000.

6. How much is 18 % of 200 ?   16 % of 300 ?

WRITTEN PROBLEMS

239. 1. A school contained 600 pupils.   88 % of them were promoted.   How many were promoted ?

2. The receipts of a store were $ 1500.   36 % of the receipts was spent for rent.   How much was paid for rent ?

3. One building is 400 ft. high.   Another building is 72 % as high.   How high is the second building ?

4. A farmer raised 1800 bu. of wheat and corn. 62 % of the total amount was wheat.

(*a*) How many bushels of wheat were raised ?

(*b*) How many bushels of corn were raised ?

5. $3500 was divided among 3 children. The first child received 45 % of the money; the second, 25 % ; and the third, the remainder. How much was paid to each child ?

6. A piano was marked for sale at $700. It was bought at a reduction of 12 %. How much was paid for it ?

7. 960 chests of tea were bought for $3.50 each. 45 % of them were sold for $5; the remainder for $5.50 each. Find the profit.

8. A hotel keeper has been buying 3000 loaves of bread a day at 3½ ct. a loaf. The price is increased 18 %. What is the daily additional cost ?

9. A clerk receives $25 a week. He saves 65 % of that amount. How much does he save in 24 wk. ?

10. Railroad tickets which formerly cost $2 are increased 26 % in price. How much more money does the company receive for carrying 3600 passengers who purchase such tickets ?
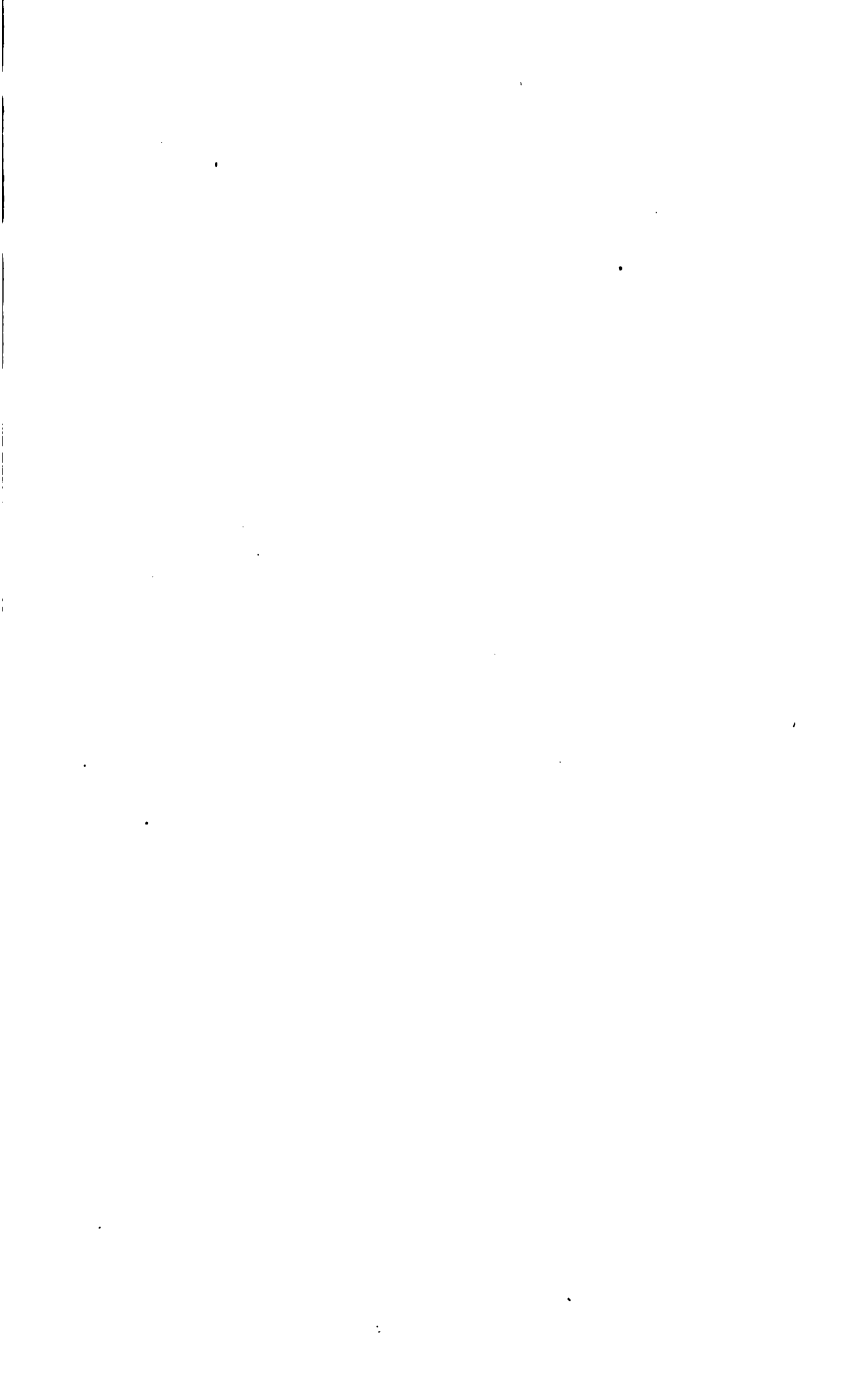
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